Partial Splitting of Longevity and Financial Risks: The Longevity Nominal Choosing Swaption

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Motivations

The insurance industry is facing specific challenges related to longevity risk

- Accurate longevity projections are delicate
- Modelling the embedded financial risk (such long term interest rate risk) remains challenging
- Important to find a suitable and efficient way to cross-hedge or to transfer part of the longevity risk to re-insurers or to financial markets

Motivations

Difficulties for launching a pure longevity product:

- No longevity market (No shared reference)
- Information asymmetry
- Modelling and pricing (evaluation in historical probability)

PURE INTEREST RATE PRODUCTS

Interest rate risk

- Insurance companies are exposed to interest rate risk
  - Assume an insurer providing pensions at a constant annuity rate k
  - If interest rates are too low, the insurer is not able to match the level of annuity
  - Moreover, low interest rates induces a higher impact of long term payments for pensions

  $\Rightarrow$ Investment in interest rate products (bonds, swaps, swaptions,...) to ensure a sufficient high interest rate
- Looking for a financial product that transfers interest rate risk while keeping longevity risk:
  - Insurance can manage longevity risk
  - Banks can manage interest rate risk

Plan

Traditional Interest Rate Risk Transfer

Plain Vanilla Swaps

An agreement on the OTC market between two parties to exchange fixed interest rate on a notional principal on specific dates for a floating interest rate, in general a Libor rate $L(., \delta)$ on the same notional.

- Issuance date = starting time T_0
- Notional amount N
- Reset/Settlement dates (tenor structure) $[\mathbf{T}] = [T_1, \cdots, T_n]$ and $\delta = T_j - T_{j-1}$ constant
- Coupon paid at T_j: $\delta L(T_{j-1}, \delta) \times N$

Plain Vanilla Swaps

Elements of Plain Vanilla Swaps

- Cash flows $[\mathbf{C}] = [N\delta L(T_0, \delta), \cdots, N\delta L(T_{n-1}, \delta)]$
  - The cash flows of the floating leg of the swap are similar to the interests of a loan with variable interest rate written on a nominal amount N.
- Price of a zero-coupon bond at t maturing at T_j, $B(t, T_j)$
  - the forward price at time t of a zero-coupon starting at T and maturing at T_j, is $B_t(T, T_j) = B(t, T_j)/B(t, T)$
- The Present Value of discounted cash flows of the floating leg, $\text{FLL}(t, T_0, [\mathbf{T}], \delta)$
- The Present Value of discounted cash flows of the fixed leg, $\text{FIL}(t, T_0, [\mathbf{T}], \delta)$
- The swap rate $S(t, T_0, [\mathbf{T}], \delta)$ determines the cash-flows of the fixed leg

Pricing Plain Vanilla Swaps

Floating Leg Value No model !

- The value at time T_0 is $\text{FLL}(T_0, T_0, [\mathbf{T}], \delta) = N(1 - B(T_0, T_n))$.
- Thus at time t, $\text{FLL}(t, T_0, [\mathbf{T}], \delta) = N(B(t, T_0) - B(t, T_n))$.

Fix Leg Value

- Given a swap rate $S(t, T_0, [\mathbf{T}], \delta)$, the value at time tl is $\text{FIL}(t, T_0, [\mathbf{T}], \delta) = NS(t, T_0, [\mathbf{T}], \delta) \sum_{i=1}^{n} \delta B(t, T_i)$.

Fair Valuation

- Swap contrat have a zero initial value, $\text{FLL}_t = \text{FIL}_t$

$$S(t, T_0, [\mathbf{T}], \delta) = \frac{B(t, T_0) - B(t, T_n)}{\sum_{i=1}^{n} \delta B(t, T_i)} = \frac{1 - B_t(T_0, T_n)}{\sum_{i=1}^{n} \delta B_t(T_0, T_i)}.$$

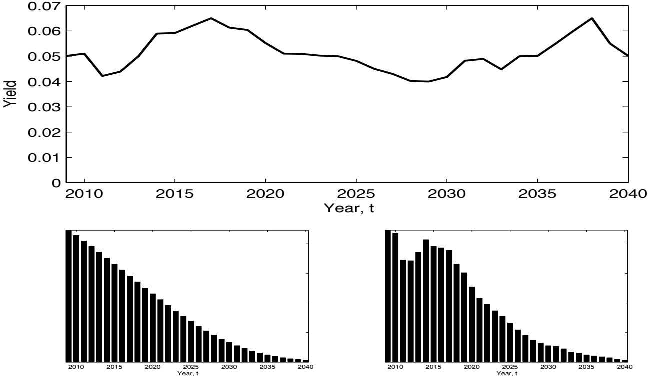
Exotic Swaps

More Exotic Swaps

- Swap with a variable nominal term structure, $[\mathbf{N}] = [N_1, \cdots, N_n]$.
- Equivalent to a family of plain vanilla swaps with different tenors and nominal
  - $[\mathbf{T}_1] = [T_1, \cdots, T_n]$ and $[\mathbf{N}_1] = [N_1, \cdots, N_n]$
  - $[\mathbf{T}_2] = [T_2, \cdots, T_n]$ and $[\mathbf{N}_2 - \mathbf{N}_1] = [N_2 - N_1, \cdots, N_2 - N_n]$
  - $\vdots$
  - $[T_n]$ and $[N_n - N_{n-1}]$.
- The exotic swap rate is

$$S(t, T_0, [\mathbf{T}], [\mathbf{N}], \delta) = \frac{\sum_{i=1}^{n} \left(N_{T_i} - N_{T_{i-1}}\right)\left(B(t, T_{i-1}) - B(t, T_n)\right)}{\delta \sum_{i=1}^{n} N_i B(t, T_i)}.$$

Traditional Interest Rate Risk Transfer



Hình : Top panel: Simulated path of the LIBOR rate $\delta L(t, \delta)$ between 2009 and 2040. Bottom panel: Cash flows of the fixed leg of swap with nominal term structure: δN_t (left) and cash flows of the floating- leg of swap with nominal term structure: $\delta L(t, \delta) N_t$ (right).

Swaptions

Swaptions

An option allowing the holder to enter some pre-specified underlying swap contract on a pre-specified date (the expiration of the swaption, denoted T, $T < T_0$). The pre-specified swap rate is the strike K.
Receiver swaption: an option that gives the buyer the right to receive the fixed leg of the swap

The pay-off of the receiver swaption $\Phi_T(T_0, [\mathbf{T}], N, K, \delta)^+$, is

$$\begin{aligned} \Phi_T(T_0, [\mathbf{T}], N, K, \delta) &= \big(K - S_T(T_0, [\mathbf{T}], \delta)\big) \sum_{i=1}^{n} N\delta B(T, T_i) \\ &= K \sum_{i=1}^{n} N\delta B(T, T_i) - \big(B(T, T_0) - B(T, T_n)\big) \end{aligned}$$

Plan

Longevity Bond

Longevity Bond (see Blake, Cairns, Dowd, 2006)

- "Principal-at-risk" type: the investor risks losing all or part of the nominal if the relevant mortality event occurs
- "Coupon-based" type: pays a coupon proportional to the number of survivors in a predetermined population (ex: birth cohort)
- Fixed or stochastic maturity (extinction of the population)

Longevity Swap

Longevity Swap

- The insurer exchanges series of nominal term structure with an investor
- The insurer pays the fixed nominal term structure $[\bar{\mathbf{L}}] = [\bar{L}_1, \cdots, \bar{L}_n]$
- The investor pays the random nominal term structure $[\tilde{\mathbf{L}}] = [\tilde{L}_1, \cdots, \tilde{L}_n]$ (embeds longevity risk)
- For all i, the fixed nominal is the historical expected value of the variable nominal + additional charge $\bar{L}_i = \left[\tilde{L}_i\right] + \rho_i$
- Risk premiums $\rho_i,\ 1 \leq i \leq n,$ include longevity risk reward and interest rate risk associated with cash flows

Longevity Swap Characteristics

- **Customized:** Based on the insurer portfolio's longevity
  - Information asymmetry
  - Very expensive
  - Legal aspects, counterparty risk...
- **Index-Based** Standardized transfers based on a *quoted* longevity index
  - Imperfect hedge: Basis risk
  - Less attractiveness

Longevity Swap

Customized transaction

- The random number $\tilde{L}_i$ represents the payments made at T_i on a run-off portfolio of retirement policies

Example

- July 2008, JP Morgan executed a customized longevity swap with a UK life insurer for a notional amount of GBP 500 millions for 40 years.
- JP Morgan entered into smaller swaps with several investors
- The investors are provided with the relevant information regarding the underlying portfolio

Longevity Swap

Index-based transaction

- The random amount $\tilde{L}_i$ is calculated as the reference survival index published at time T_i, applied to a predetermined nominal amount

Example

- In January 2008, JP Morgan with the pension insurer Lucida for a notional amount of GBP 100 millions for 10 years
- LifeMetrics index for England and Wales as underlying reference index
- Lucida kept basis risk

Longevity Risk Transfer

Motivation

Due heavy costs of longevity covers and cross-hedging possibilities with mortality risk, the insurer might prefer:

- To keep the longevity risk
- Hedge the against interest rate risk, written on the random nominals

Allows the insurer to re-adjust the nominal structure

Plan

Longevity Nominal Chooser Swaption

Description

An option allowing the holder to enter some pre-specified underlying swap contract on a pre-specified date (the expiration of the swaption, i.e. $T < T_0$), with the possibility of choosing the nominal term structure at time T.

1. At the issuance date T_0: The insurer estimates two nominal term structures (e.g. quantile estimated), denoted $[\mathbf{N_T^+}] = [N_{1,T}^+, \cdots, N_{n,T}^+]$ and $[\mathbf{N_T^-}] = [N_{1,T}^-, \cdots, N_{n,T}^-]$
2. At the expiration date T: The insurer has in addition the right to revise the nominal structure by choosing a parameter $\alpha_T \in [0, 1]$, i.e. $[\mathbf{N_T^\alpha}] = \alpha_T[\mathbf{N_T^-}] + (1 - \alpha_T)[\mathbf{N_T^+}]$

Longevity Nominal Chooser Swaption

Product Design

- Longevity forecasting: Estimate of the extreme series of nominal structure
  - Takes into account portfolio heterogeneity: socio-professional category, marital status, education level, gender.
- Maturity: T_n must be the time at which all policyholders have died
  - In practice, the time where the number of policyholders still alive is smaller than some threshold;
- Pricing: The product pricing is operated on the basis of the worst-case scenario

Longevity Nominal Chooser Swaption

Pricing

- The price of the swaption with nominal term structure $[\mathbf{N}^{\alpha}_{\mathbf{T}}] = \alpha_T[\mathbf{N}^{-}_{\mathbf{T}}] + (1-\alpha_T)[\mathbf{N}^{+}_{\mathbf{T}}]$ is given by

$$\Phi_T(T_0, [\mathbf{T}], [\mathbf{N}^{\alpha}], K, \delta)$$
$$= \big(K\sum_{i=1}^{n} N_i^{\alpha}\delta B(T,T_i) - \sum_{i=1}^{n}\delta(N_i^{\alpha} - N_{i-1}^{\alpha})\big(B(T,T_{i-1}) - B(T,T_n)\big)\big).$$

- The product pricing is operated on the basis of the worst-case scenario, corresponding to the maximum of the pay-offs:

$$\max_{0\leq\alpha\leq 1}\Phi_T(T_0, [\mathbf{T}], [\mathbf{N}^{\alpha}_{\mathbf{T}}], \mathbf{K}, \delta)^{+}$$

Plan

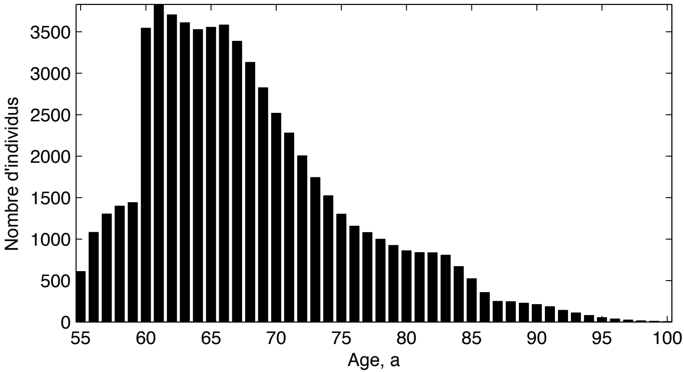
Quantitative analysis

Modelling

- CBD type model for death probabilities
  - Provide trajectories of death probabilities $q(a, t, x)$ by age, year, and characteristics
- Dynamic population model for policyholders evolution
  - Takes into account the characteristics of each individual in the portfolio
  - Provides scenarios of the evolution of the population itself
- Heath-Jarrow-Morton two-factor model
  - Calibrated on a set of relevant swaption prices and on the European yield curve

Quantitative analysis

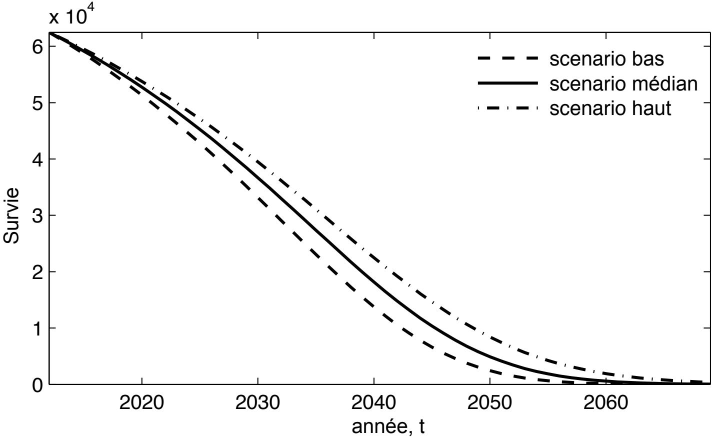
Insurance portfolio with 62482 French, male policyholders in 2012 We suppose that $k = 4.5\%$



Hình : Age structure of policyholders

Quantitative analysis

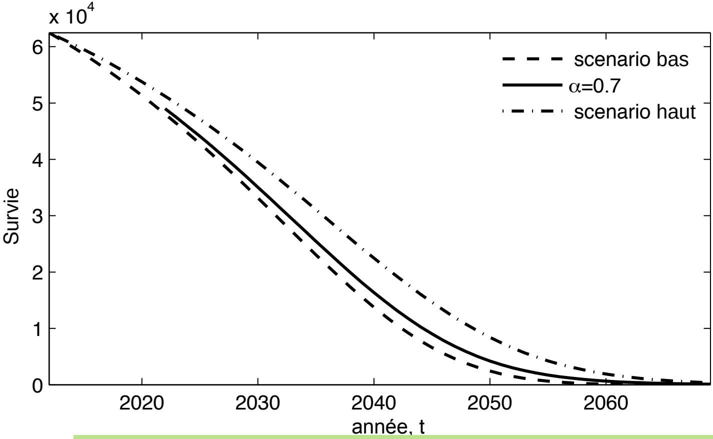
Scenarios of the real-world portfolio evolution



Quantitative analysis

An example of possible choice of the parameter α_T in 2022

$N_T^- = 48049$, $N_T^+ = 51223$, then if $N_T = 49000$ we can fix $\alpha_T = (N_T^+ - N_T)/(N_T^+ - N_t) \approx 0.7$



Quantitative analysis

Impact of Swap Rates Correlation

- Evolution of price as a function of correlation between swap rates with maturities $10Y$ and $12Y$.

Correlation swaps		**Price**		**Price of**	**Cost on**
$\mathbf{10Y/12Y}$	$\boldsymbol{\alpha = 0}$	$\boldsymbol{\alpha = 0.5}$	$\boldsymbol{\alpha = 1}$	**product**	**annuity**
99.6%	1098 bp	983 bp	871 bp	1109 bp	0.764%
99.3%	1072 bp	965 bp	860 bp	1086 bp	0.749%
99%	1046 bp	945 bp	849 bp	1066 bp	0.735%
98.7%	1018 bp	925 bp	837 bp	1043 bp	0.719%
98.3%	988 bp	903 bp	824 bp	1019 bp	0.702%
97.8%	957 bp	879 bp	809 bp	994 bp	0.685%
97.3%	924 bp	854 bp	793 bp	967 bp	0.667%

- Price of the switch option, $1109 - 1098 = 11$bps

Conclusion

- Interest rate could be hedged using an exotic structured product and thus allows the insurer to keep longevity risk
- The price of such a product is strongly dependent on the correlation of the swap correlation, and other factors not exposed in this presentation: The strike, for instance.
- We considered only with fixed annuity rates
- For a more *realistic* example, one should take into consideration the inflation
- Lay the emphasis on counterparty risk given the long-run maturities
- Throughout the paper we implicitly assumed that longevity risk and interest rates were independent