Return Decomposition over the Business Cycle

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Introduction

- Stock prices depend on investors' expectations about future cash flows and discount rates and, thus, changes in investors' expectations about future cash flows or discount rates move stock prices.
- A natural question: Is the observed variation in stock prices mostly due to changes in investors' expectations about future cash flows or discount rates?
- Although this is an empirical question, it also has important implications for theoretical asset pricing.
  - It would allow us to understand how financial markets work.
  - It would provide empirical guidance in modeling stock returns.

Introduction

- To address this issue, Campbell and Shiller (1988a) suggest decomposing stock returns into two components:
  1. changes in investors' expectations about discounted sum of future excess returns, or equivalently future discount rates, which is commonly referred to as the discount rate news
  2. changes in investors' expectations about discounted sum of future dividend growth rates, or equivalently future cash flows, which is commonly referred to as the cash flow news.
- This simple decomposition in turn implies that the variance of stock returns can be decomposed into the variance of cash flow and discount rate news and the covariance between the two components.
- Based on this decomposition, they propose analyzing the relative importance of each component in determining the observed variation in stock prices by considering their contribution to the unconditional variance of stock returns.

Introduction

- Given that neither discount rate nor cash flow news can be directly observed, one has to find empirical proxies to analyze their relative contribution to the observed variation in stock prices.

- The standard approach in the literature is to

  1. model the short-run dynamics of expected returns in a vector autoregressive (VAR) system
  2. obtain an empirical proxy for the discount rate news based on forecasts from the estimated VAR system
  3. back out cash flow news based on the relation between returns and its components.

Introduction

- The standard approach has several advantages.
    1. One needs to understand only the short-run dynamics of expected returns and not that of cash flows, which can be relatively difficult to model.
    2. It has been easier to forecast returns than dividends, at least in the last 50 years.
    3. It is a very straightforward and easy-to-implement approach as it only requires the estimation of a simple VAR.
- Hence, it is not surprising to find a large literature implementing the standard approach to answer different questions in finance, macroeconomics and accounting.
- However, the standard approach depends heavily on the predictability of returns. Given the growing empirical evidence against the predictability of returns, it has also recently come under some criticism.

Motivation

- Most studies in the literature focus on the decomposition of the unconditional variance of stock returns based on the standard approach with linear VAR models and constant parameters.
- However, there is growing empirical evidence that both variances and relations between variables in financial markets are time-varying.
  1. It is a well-known empirical fact that the conditional variances of stock returns and most of the standard predictor variables are time-varying and vary with changing market conditions. For example, most financial variables, including but not limited to stock returns, tend to be much more volatile in recessions than expansions.
  2. There is growing empirical evidence that the relation between stock returns and predictive variables is also time-varying and depends on underlying business and economic conditions (see Dangl and Halling (2011) and references therein.).

Motivation

- These empirical facts in turn suggest that
  1. the decomposition of returns based on an approach that captures the time-varying nature of parameters and variances might be completely different than that based on the standard approach
  2. the decomposition of conditional variances can be completely different than that of unconditional variances and it might also be changing over time as the economy and financial markets go through periods of tranquility and turbulence.

Summary

- Motivated by these empirical facts, we generalize the standard approach to a framework where we model the short-run dynamics of returns and predictive variables in a Markov regime switching vector autoregressive model (MSVAR) with both the VAR parameters and residual variance matrix switching between different values based on the underlying state of the economy.
- In this framework, we show that the conditional variances of cash flow and discount rate news as well as their conditional covariance can be expressed in closed-from when the state variable is observable and can be calculated numerically based on simulations otherwise.
- Based on this time-varying approach, we analyze the decomposition of the conditional variance of returns on the S&P 500 index over the business cycle.

Summary

- The cash flow news is relatively more important than discount rate news in determining the conditional variance of returns in expansions.
- The conditional variances of returns and its components increase in recessions.
- However, the conditional variance of returns increases more than that of cash flow news and the discount rate news becomes relatively more important than cash flow news in determining the conditional variance of returns in recessions.
- In contrast to the standard Campbell and Shiller approach, cash flow news is more important than discount rate news in determining the unconditional variance of returns when we allow parameters and variances to vary over the business cycle.
- We show that these results are broadly consistent with the implications of a stylized asset pricing model in which the growth rates of dividends and consumption take on different values depending on the underlying state of the economy.

Part I

Some Preliminaries on Return Decomposition

Return Decomposition

- The Campbell and Shiller decomposition approach starts with the definition of the log stock return, r_{t+1}

$$\begin{aligned} r_{t+1} &= \log(P_{t+1} + D_{t+1}) - \log(P_t) \\ r_{t+1} &= p_{t+1} - p_t + \log(1 + \exp(d_{t+1} - p_{t+1})) \end{aligned}$$

- The last term on the right-hand side is a nonlinear function of the log dividend-price ratio, $f(d_{t+1} - p_{t+1})$.
- Like any nonlinear function, $f(x_{t+1})$, it can be approximated around the mean of x_{t+1}, $\bar{x}$, using a first-order Taylor expansion:

$$f(x_{t+1}) \approx f(\bar{x}) + f'(\bar{x})(x_{t+1} - \bar{x})$$

Return Decomposition

- Substituting the approximation of log dividend-price ratio, $f(d_{t+1} - p_{t+1})$, recursively into the definition of the log stock return and taking conditional expectations under a transversality condition

$$p_t = \frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty} \rho^j [(1-\rho) d_{t+1+j} - r_{t+1+j}]\right]$$

  where ρ and k are parameters of linearization defined by $\rho = 1/(1+\exp(\overline{d-p}))$, where $(\overline{d-p})$ is the average log dividend-price ratio, and $k = -\log(\rho) - (1-\rho)\log(1/\rho - 1)$.

- This can be considered as a dynamic generalization of the Gordon formula for a stock price with constant required returns and dividend growth.

Return Decomposition

- This relation between prices, returns and dividends is just an accounting identity, i.e. there is no economic model behind it.
- High prices must eventually be followed by high future dividends, low future returns, or some combination of the two.
- And, the investors' expectations must be consistent with this, so high prices must be associated with high expected future dividends, low expected future returns, or some combination of the two.

Return Decomposition

- One can also write the same equation for returns,

$$
\begin{aligned}
r_{t+1} - E_t[r_{t+1}] &= E_{t+1}\left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j}\right] - E_t\left[\sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j}\right] \\
&- \left(E_{t+1}\left[\sum_{j=1}^{\infty} \rho^j r_{t+1+j}\right] - E_t\left[\sum_{j=1}^{\infty} \rho^j r_{t+1+j}\right]\right) \\
&= CF_{t+1} - DR_{t+1}
\end{aligned}
$$

- Similarly, high returns must be associated with upward revisions in expected future dividends, downward revisions in expected future returns, or some combination of the two.

Part II

The Standard Approach

The Standard Approach - Introduction

- To operationalize this, one needs to choose a value for ρ and a forecasting model for returns with a set of predictor variables.

- The standard approach in the literature is to
  1. Estimate a VAR(1) model for stock returns and other state variables that might have predictive power for stock returns.
  2. Estimate the current discount rate news as the change in the expected future stock returns based on the forecasts obtained from the estimated VAR(1) system.
  3. Back out the current cash flow news as the difference between the current unexpected return and discount rate news.

The Standard Approach - Introduction

- The unconditional variance of unexpected return can then be decomposed into three components:
  - the variance of cash flow news
  - the variance of discount rate news
  - their covariance

- The relative importance of each component in determining the historical behavior of returns can then be analyzed as the percentage of total unconditional variance of unexpected returns that can be attributed to the variance of that component.

The Standard Approach - Forecasting Model

- The standard forecasting model is a constant parameter VAR:

$$\mathbf{X}_{t+1} = \alpha + \mathbf{A}\mathbf{X}_t + \epsilon_{t+1}$$

where $\mathbf{X}_{t+1} = [r_{t+1}, \mathbf{z}'_{t+1}]'$.

- Note the following important fact:

$$E_t[\mathbf{X}_{t+\tau}] = (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{I} - \mathbf{A}^{\tau})\alpha + \mathbf{A}^{\tau}\mathbf{X}_t$$

The Standard Approach - Return Decomposition

- Having estimated the VAR, the unexpected return at time $t+1$,

$$r^*_{t+1} \equiv r_{t+1} - E_t[r_{t+1}] = \mathbf{e}_1' \hat{\epsilon}_{t+1}$$

can be decomposed

  - into discount rate news

$$\begin{aligned} DR_{t+1} &= E_{t+1}\left[\sum_{j=1}^{\infty} \rho^j r_{t+1+j}\right] - E_t\left[\sum_{j=1}^{\infty} \rho^j r_{t+1+j}\right] \\ &= \mathbf{e}_1' \sum_{j=1}^{\infty} \rho^j \hat{\mathbf{A}}^j \hat{\epsilon}_{t+1} = \mathbf{e}_1' \rho \hat{\mathbf{A}} (\mathbf{I} - \rho \hat{\mathbf{A}})^{-1} \hat{\epsilon}_{t+1} \\ &= \mathbf{e}_1' (\mathbf{I} - \rho \hat{\mathbf{A}})^{-1} \rho \hat{\mathbf{A}} (\mathbf{X}_{t+1} - \hat{\alpha} - \hat{\mathbf{A}} \mathbf{X}_t) \end{aligned}$$

  - into cash flow news

$$\begin{aligned} CF_{t+1} &= (r_{t+1} - E_t[r_{t+1}]) + DR_{t+1} \\ &= \mathbf{e}_1' (\mathbf{I} + \rho \hat{\mathbf{A}} (\mathbf{I} - \rho \hat{\mathbf{A}})^{-1}) \hat{\epsilon}_{t+1} \\ &= \mathbf{e}_1' (\mathbf{I} + \rho \hat{\mathbf{A}} (\mathbf{I} - \rho \hat{\mathbf{A}})^{-1}) (\mathbf{X}_{t+1} - \hat{\alpha} - \hat{\mathbf{A}} \mathbf{X}_t) \end{aligned}$$

The Standard Approach - Variance Decomposition

- Then, the unconditional variance of unexpected returns can be decomposed into
  - the unconditional variance of cash flow news

$$var(CF_{t+1}) = (\mathbf{e}_1'(\mathbf{I} + \rho\hat{\mathbf{A}}(\mathbf{I} - \rho\hat{\mathbf{A}})^{-1}))\hat{\mathbf{\Sigma}}(\mathbf{e}_1'(\mathbf{I} + \rho\hat{\mathbf{A}}(\mathbf{I} - \rho\hat{\mathbf{A}})^{-1}))'$$

  - the unconditional variance of discount rate news

$$var(DR_{t+1}) = (\mathbf{e}_1'\rho\hat{\mathbf{A}}(\mathbf{I} - \rho\hat{\mathbf{A}})^{-1})\hat{\mathbf{\Sigma}}(\mathbf{e}_1'\rho\hat{\mathbf{A}}(\mathbf{I} - \rho\hat{\mathbf{A}})^{-1})'$$

  - the unconditional covariance between the two

$$cov(DR_{t+1}, CF_{t+1}) = (\mathbf{e}_1'\rho\hat{\mathbf{A}}(\mathbf{I} - \rho\hat{\mathbf{A}})^{-1})\hat{\mathbf{\Sigma}}(\mathbf{e}_1'(\mathbf{I} + \rho\hat{\mathbf{A}}(\mathbf{I} - \rho\hat{\mathbf{A}})^{-1}))'$$

The Standard Approach - Data

- We are interested in the relative importance of each component in determining the returns on an aggregate index.
- To this end, we analyze the variance decomposition of monthly excess returns on the S&P 500 index between January 1960 and December 2010.
- To operationalize this, one needs to choose a value for ρ, a set of predictor variables and a forecasting model for returns.
- We fix ρ at 0.997 in monthly data which correspond to an annual average dividend-price ratio of around 4%.
- We use term spread (tms), dividend yield (dy) and value spread (vs) as predictor variables.

The Standard Approach - Results

Table : Unconditional Variance Decomposition of Returns

	Value (Ratio)
$var(CF)$	5.43 (28.99%)
$var(DR)$	8.14 (43.44%)
$-2cov(CF, DR)$	5.17 (27.57%)
$var(r)$	18.74 (100.00%)

Part III

Time Varying Relative Importance

Time Varying Predictive Relations

Table : Whole Sample

	α	r_{t-1}	tms_{t-1}	dy_{t-1}	vs_{t-1}	$\bar{R}^2$
r_t	0.5469	0.0486	0.2438**	0.2331	-0.8614	0.68%
tms_t	-0.0955	0.0039	0.9558***	0.0097	0.0988	91.07%
dy_t	0.1047	-0.0019	-0.0114***	0.9875***	-0.0323	98.24%
vs_t	0.1058***	-0.0003	-0.0002	-0.0021	0.9331***	88.11%

Table : Expansions

	α	r_{t-1}	tms_{t-1}	dy_{t-1}	vs_{t-1}	$\bar{R}^2$
r_t	0.4647	-0.0448	0.1460	0.1303	-0.3119	0.40%
tms_t	0.1597	0.0050	0.9758***	-0.0306*	-0.0444	94.79%
dy_t	0.0923	0.0007	-0.0069**	0.9917***	-0.0422	99.39%
vs_t	0.1115***	-0.0005	-0.0022*	-0.0016	0.9305***	99.44%

Table : Recessions

	α	r_{t-1}	tms_{t-1}	dy_{t-1}	vs_{t-1}	$\bar{R}^2$
r_t	0.4261	0.1699***	0.7463***	1.0404***	-1.8159	10.71%
tms_t	0.0287**	0.0143***	0.8470***	0.0270	0.7336***	90.11%
dy_t	0.3204	-0.0051***	-0.0377***	0.9621***	0.0494	99.59%
vs_t	0.0000***	-0.0003	0.0127***	-0.0051**	0.8605***	99.82%

Time Varying Covariances

Table : Whole Sample

	r_t	tms_t	dy_t	vs_t
r_t	18.7419	0.0385	-0.6079	0.0416
tms_t	0.0385	0.1974	-0.0025	-0.0014
dy_t	-0.6079	-0.0025	0.0227	-0.0014
vs_t	0.0416	-0.0014	-0.0014	0.0024

Table : Expansions

	r_t	tms_t	dy_t	vs_t
r_t	15.3599	-0.0623	-0.4523	0.0401
tms_t	-0.0623	0.1409	0.0035	-0.0016
dy_t	-0.4523	0.0035	0.0153	-0.0012
vs_t	0.0401	-0.0016	-0.0012	0.0021

Table : Recessions

	r_t	tms_t	dy_t	vs_t
r_t	31.9502	0.8037	-1.2739	0.0391
tms_t	0.8037	0.4029	-0.0415	0.0025
dy_t	-1.2739	-0.0415	0.0561	-0.0020
vs_t	0.0391	0.0025	-0.0020	0.0033

Return Decomposition Based on the Standard Approach Under Alternative Assumptions

Table : Time-Varying VAR Parameters and Constant Residual Variance

	Expansions	Recessions
$var(CF)$	34.87 (192.69%)	4.37 (24.15%)
$var(DR)$	9.66 (53.38%)	17.54 (96.90%)
$-2cov(CF, DR)$	-26.43 (-146.07%)	-3.81 (-21.05%)
$var(r)$	18.10 (100.00%)	18.10 (100.00%)

Table : Constant VAR Parameters and Time-Varying Residual Variance

	Expansions	Recessions
$var(CF)$	4.98 (31.98%)	7.75 (22.25%)
$var(DR)$	6.10 (39.22%)	18.45 (52.98%)
$-2cov(CF, DR)$	4.48 (28.80%)	8.63 (24.77%)
$var(r)$	15.56 (100.00%)	34.83 (100.00%)

Table : The Joint Effect

	Expansions	Recessions
$var(CF)$	25.90 (168.64%)	9.40 (29.42%)
$var(DR)$	6.90 (44.93%)	47.05 (147.26%)
$-2cov(CF, DR)$	-17.44 (-113.57%)	-24.50 (-76.68%)
$var(r)$	15.36 (100.00%)	31.95 (100.00%)

Part IV

Time Varying Return Decomposition

A MSVAR Forecasting Model

- A flexible and parametric approach to capture these effects is to model the dynamics of returns and predictive variables in a Markov regime switching vector autoregression (MSVAR) as follows:

$$\mathbf{X}_{t+1} = \alpha_{S_{t+1}} + \mathbf{A}_{S_{t+1}} \mathbf{X}_t + \epsilon_{t+1}$$

- The state variable S_t follows a first order M-state Markov chain with transition probability matrix $\mathbf{Q}$ whose ij^{th} element $q_{i,j} = Prob(S_{t+1} = j | S_t = i)$.

Lemma

$$E_t[\mathbf{X}_{t+\tau}] = (\mathbf{1}_M \otimes \mathbf{I}_N)' \left(\mathbf{f}_1(\tau)(\mathbf{\Pi}_t \otimes \mathbf{1}_N) + \mathbf{f}_2(\tau)(\mathbf{\Pi}_t \otimes \mathbf{I}_N)\mathbf{X}_t \right)$$

where $\mathbf{\Pi}_t = [Prob(S_t = 1|\mathcal{F}_t), Prob(S_t = 2|\mathcal{F}_t), \ldots, Prob(S_t = M|\mathcal{F}_t)]'$.

Return Decomposition

Proposition

The unexpected return on the risky asset in period $t+1$ can be expressed as follows:

$$r^*_{t+1} = \mathbf{e}_1' \left(\mathbf{X}_{t+1} - (\mathbf{1}_M \otimes \mathbf{I}_N)' (\mathbf{f}_1(1)(\boldsymbol{\Pi}_t \otimes \mathbf{1}_N) + \mathbf{f}_2(1)(\boldsymbol{\Pi}_t \otimes \mathbf{X}_t)) \right)$$

and can be decomposed into cash flow and discount rate news:

$$\begin{aligned} DR_{t+1} &= \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'[\mathbf{B}_{1,1}(\boldsymbol{\Pi}_{t+1} \otimes \mathbf{1}_N) + \mathbf{B}_{2,1}(\boldsymbol{\Pi}_{t+1} \otimes \mathbf{X}_{t+1}) \\ &- (\mathbf{B}_{1,2}(\boldsymbol{\Pi}_t \otimes \mathbf{1}_N) + \mathbf{B}_{2,2}(\boldsymbol{\Pi}_t \otimes \mathbf{X}_t))] \end{aligned}$$

and

$$\begin{aligned} CF_{t+1} &= \mathbf{e}_1'\mathbf{X}_{t+1} + \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'[\mathbf{B}_{1,1}(\boldsymbol{\Pi}_{t+1} \otimes \mathbf{1}_N) + \mathbf{B}_{2,1}(\boldsymbol{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})] \\ &- \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'[(\mathbf{f}_1(1) + \mathbf{B}_{1,2})(\boldsymbol{\Pi}_t \otimes \mathbf{1}_N) + (\mathbf{f}_2(1) + \mathbf{B}_{2,2})(\boldsymbol{\Pi}_t \otimes \mathbf{X}_t)] \end{aligned}$$

Unconditional Variance Decomposition based on the Time-Varying Approach over the Business Cycle

	Value (Ratio)
$var(CF)$	8.49 (46.41%)
$var(DR)$	7.25 (39.64%)
$-2cov(CF, DR)$	2.55 (13.95%)
$var(r)$	18.29 (100.00%)

Conditional Variance Decomposition

Proposition

The conditional variance of returns is given by

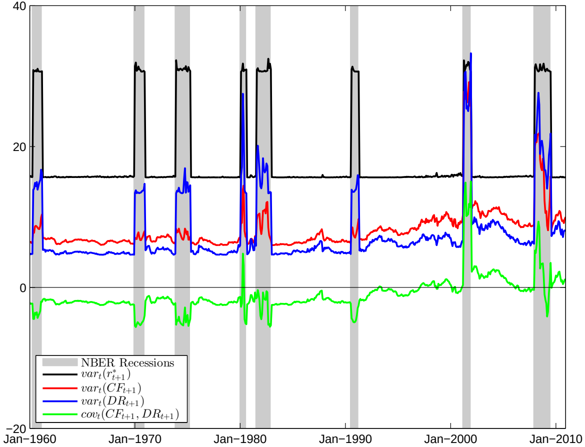
$$var_t(r^*_{t+1}) = \mathbf{e}_1'(\mathbf{1}_M \otimes \mathbf{I}_N)'(\mathbf{\Omega}_t - \mathbf{Z}_t\mathbf{Z}_t')(\mathbf{1}_M \otimes \mathbf{I}_N)\mathbf{e}_1$$

$\mathbf{\Omega}_{i,t} = (\alpha_i\alpha_i' + \alpha_i(\mathbf{A}_i\mathbf{X}_t)' + (\mathbf{A}_i\mathbf{X}_t)\alpha_i' + (\mathbf{A}_i\mathbf{X}_t)(\mathbf{A}_i\mathbf{X}_t)' + \Sigma_i)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t)$ *and* $\mathbf{Z}_t = [\mathbf{Z}_{1,t}', \ldots, \mathbf{Z}_{M,t}']'$ *with* $\mathbf{Z}_{i,t} = (\alpha_i + \mathbf{A}_i\mathbf{X}_t)(\mathbf{e}_i'\mathbf{Q}'\mathbf{\Pi}_t)$.

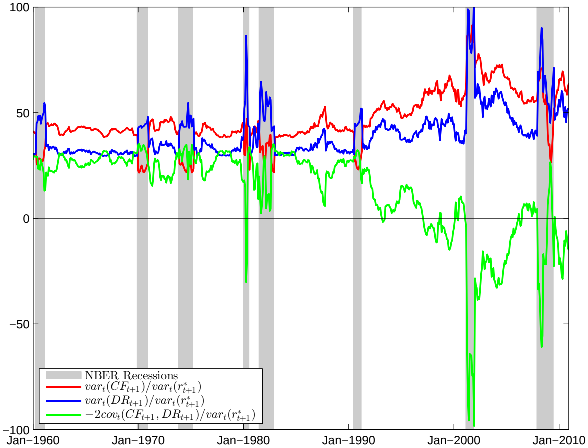
The conditional variances of discount rate and cash flow news and their conditional covariance can be written in closed from as functions of $var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)$, $var_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})$, $cov_t(\mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})$, $cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{1}_N)$ *and* $cov_t(\mathbf{X}_{t+1}, \mathbf{\Pi}_{t+1} \otimes \mathbf{X}_{t+1})$.

Furthermore, these conditional quantities can be calculated analytically when the state variable is assumed observable and numerically when the state variable is assumed latent.

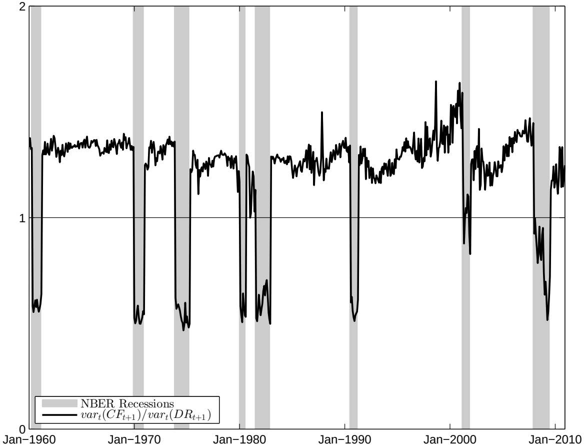
Conditional Variances over the Business Cycle



Relative Importance over the Business Cycle



Ratio of Conditional Variances of Cash Flows and Discount News



Robustness Checks

- Chen and Zhao (2009) show that the empirical results based on the standard return decomposition approach tend to be sensitive to the set of state variables and the sample period.
- To this end, we also analyze the robustness of our main empirical results based on the time-varying approach and find that they are mostly robust to
  1. using a longer sample period between June 1927 and December 2010
  2. using the first four principal components of a large number of known predictor variables as an alternative set of state variables
  3. using an alternative definition of the business cycle based on the smoothed state probabilities obtained from the estimation of a two-state Markov regime switching model for the log growth rate of monthly industrial production index.

Part V

Implications of a Stylized Asset Pricing Model

Return Decomposition in a Structural Framework with Constant Expected Returns

Assumptions

- Distributional Assumption:

$$\begin{bmatrix} \Delta c_{t+1} \\ \Delta d_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_c \\ \mu_d \end{bmatrix} + \begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{d,t} \end{bmatrix}, \qquad \begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{d,t} \end{bmatrix} \sim N(\mathbf{0}, \mathbf{\Sigma})$$

- General or Partial Equilibrium
- Power or EZW Utility

Implications

- Price-Dividend Ratio: $\frac{P_{t+1}}{D_{t+1}} = \lambda$
- Unexpected Log Return: $r_{t+1} - E_t[r_{t+1}] = \Delta d_{t+1} - \mu_d$
- Cash Flow Component: $CF_{t+1} = \Delta d_{t+1} - \mu_d$
- Discount Rate Component: $DR_{t+1} = 0$

Return Decomposition in a Structural Framework with Time-Varying Expected Returns

Assumptions

- Distributional Assumption:

$$\begin{bmatrix} \Delta c_{t+1} \\ \Delta d_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_{c,S_{t+1}} \\ \mu_{d,S_{t+1}} \end{bmatrix} + \begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{d,t} \end{bmatrix}, \qquad \begin{bmatrix} \varepsilon_{c,t} \\ \varepsilon_{d,t} \end{bmatrix} \sim N(\mathbf{0}, \mathbf{\Sigma}_{S_{t+1}})$$

Examples

- PE and Power Utility: Cecchetti et al. (1993), Santos and Veronesi (2006)
- GE and Power Utility: David (1997), Veronesi (1999, 2000)
- PE and EZW Utility: Bansal and Yaron (2004), Calvet and Fisher (2007), Garcia et al. (2008)
- GE and EZW Utility: Lettau et al. (2008)

Return Decomposition in a Structural Framework with Time-Varying Expected Returns

- We focus on the partial equilibrium model with power utility since it allows for closed-form solutions.
- Specifically, we consider a pure exchange economy (Lucas(1978)) in discrete time where the preferences of a representative investor are represented by a constant relative risk aversion utility over consumption,

$$U(C_t) = \begin{cases} \frac{C_t^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(C_t) & \text{if } \gamma = 1 \end{cases} \quad (1)$$

  where C_t denotes investors' consumption in period t and γ is his coefficient of relative risk aversion.
- Investors' opportunity set consists of a risky asset whose supply is fixed and normalized to one and a risk-free asset.
- We assume that investors have access to implicit labor income.

Return Decomposition in a Structural Framework with Time-Varying Expected Returns

Solving the Model

- Investor's beliefs about the state variable:

$$\pi_{j,t} = \frac{\phi(\mathbf{y}_t, \boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)\tilde{\pi}_{j,t}}{\sum_{i=1}^{N} \phi(\mathbf{y}_t, \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)\tilde{\pi}_{i,t}} \qquad \text{for } j = 1, 2, \ldots, N.$$

where $\tilde{\pi}_{j,t} = \sum_{i=1}^{N} \pi_{i,t-1} q_{ij}$ and $\phi(x, \boldsymbol{\mu}, \boldsymbol{\Sigma})$ is the multivariate normal density function with mean $\boldsymbol{\mu}$ and variance matrix $\boldsymbol{\Sigma}$.

- The price-dividend ratio

$$\lambda_j = [(\mathbf{I} - \mathbf{QH})^{-1}\mathbf{QG}]_j > 0 \quad \text{for } j = 1, 2, \ldots, N$$

where the operator $[\cdot]_j$ refers to the j^{th} element of a vector. $\mathbf{I}$ is the $N \times N$ identity matrix and $\mathbf{Q}$ is the transition probability matrix. $\mathbf{G}$ is a $N \times 1$ vector and $\mathbf{H}$ is a $N \times N$ diagonal matrix.

Return Decomposition in a Structural Framework with Time-Varying Expected Returns

Implications

- Price-Dividend Ratio:
  $\frac{P_{t+1}}{D_{t+1}} = \boldsymbol{\lambda}'\boldsymbol{\Pi}_{t+1}$
  - λ_i for $i = 1, \cdots, N$ is the price-dividend ratio if $S_{t+1} = i$.
  - $\Pi_{i,t+1} = Prob(S_{t+1} = i | \mathcal{F}_{t+1})$
- Unexpected Log Return:
  $r_{t+1} - E_t[r_{t+1}] = \rho\boldsymbol{\lambda}'(\boldsymbol{\Pi}_{t+1} - \mathbf{Q}'\boldsymbol{\Pi}_t) + \Delta d_{t+1} - \bar{\mu}_{d,t}$
  - $\bar{\mu}_{d,t} = \boldsymbol{\mu}_d'\mathbf{Q}'\boldsymbol{\Pi}_t$
  - $\rho = 1/(1+\bar{\lambda})$ and $\bar{\lambda} = \boldsymbol{\lambda}'\bar{\boldsymbol{\Pi}}$
- Cash Flow Component:
  $CF_{t+1} = \boldsymbol{\mu}_d'(\mathbf{I} - (1-\rho)\mathbf{Q}')^{-1}(1-\rho)\mathbf{Q}'(\boldsymbol{\Pi}_{t+1} - \mathbf{Q}'\boldsymbol{\Pi}_t) + \Delta d_{t+1} - \bar{\mu}_{d,t}$
- Discount Rate Component:
  $DR_{t+1} = (\boldsymbol{\mu}_d'(\mathbf{I} - (1-\rho)\mathbf{Q}')^{-1}(1-\rho)\mathbf{Q}' - \rho\boldsymbol{\lambda}')(\boldsymbol{\Pi}_{t+1} - \mathbf{Q}'\boldsymbol{\Pi}_t)$

Return Decomposition in a Structural Framework with Time-Varying Expected Returns

Cond. Variance of the Unexpected Returns

$$\begin{aligned} var_t(r_{t+1}) &= \rho^2 \boldsymbol{\lambda}' var_t(\boldsymbol{\Pi}_{t+1})\boldsymbol{\lambda} + var_t(\Delta d_{t+1}) + 2\rho\boldsymbol{\lambda}' cov_t(\Delta d_{t+1}, \boldsymbol{\Pi}_{t+1}) \\ &= var_t(CF_{t+1}) + var_t(DR_{t+1}) - 2cov_t(CF_{t+1}, DR_{t+1}) \end{aligned}$$

Conditional Variance Decomposition

- Cond. Variance of the CF Component ($var_t(CF_{t+1})$)
  $\mathbf{m}_d var_t(\boldsymbol{\Pi}_{t+1})\mathbf{m}_d' + var_t(\Delta d_{t+1}) + 2\mathbf{m}_d cov_t(\Delta d_{t+1}, \boldsymbol{\Pi}_{t+1})$
- Cond. Variance of the DR Component ($var_t(DR_{t+1})$)
  $(\mathbf{m}_d - \rho\boldsymbol{\lambda}')var_t(\boldsymbol{\Pi}_{t+1})(\mathbf{m}_d - \rho\boldsymbol{\lambda}')'$
- Cond. Covariance of the CF and DR component ($cov_t(CF_{t+1}, DR_{t+1})$)
  $\mathbf{m}_d var_t(\boldsymbol{\Pi}_{t+1})(\mathbf{m}_d - \rho\boldsymbol{\lambda}')' + (\mathbf{m}_d - \rho\boldsymbol{\lambda}')cov_t(\Delta d_{t+1}, \boldsymbol{\Pi}_{t+1})$

$\mathbf{m}_d = \boldsymbol{\mu}_d'(\mathbf{I} - (1-\rho)\mathbf{Q}')^{-1}(1-\rho)\mathbf{Q}'$

Calibration and Simulation

- We simulate our model at monthly frequency for a total of 612 observations which corresponds to the number of monthly observations for the period between 1960 and 2010.
- Rather than simulating the state variable, we assume that it is observable and correspond to expansions ($S_t = 1$) and recessions ($S_t = 2$) as defined by the NBER between 1960 and 2010.
- We also calibrate $\mathbf{Q}$ to match the monthly transition probabilities the NBER business cycles between 1960 and 2010.
- To calibrate the parameters of the consumption and dividend processes, similar to Bansal and Yaron (2004), we use annual data on real per-capita personal consumption expenditures on nondurables and services and real dividends paid on the S&P 500 index to proxy for dividends between 1929 and 2010.
- Finally, we assume a monthly time impatience parameter of 0.9957, corresponding to an annual value of 0.95, and a coefficient of relative risk aversion of 7.5.

Calibrated Model Parameters

Parameter	Value
γ	7.5
β	0.9957

(a) Utility Specification

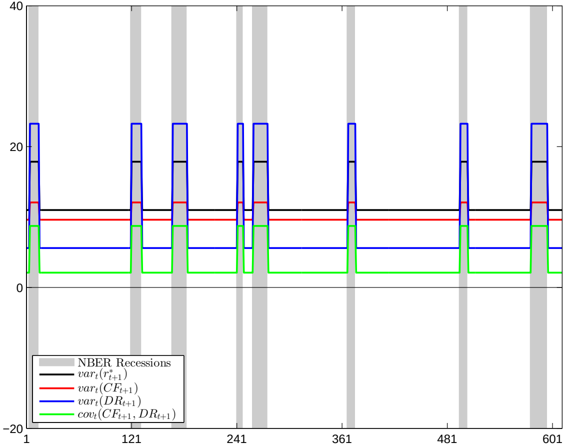
Parameter	$i = 1$	$i = 2$
$\mu_{d,i}$	0.204%	-0.556%
$\mu_{c,i}$	0.297%	0.008%
$\sigma_{d,i}$	2.972%	2.972%
$\sigma_{c,i}$	0.693%	0.693%
$\rho_{cd,i}$	0.391	0.391
q_{ii}	0.983	0.925

(b) Dividend and Consumption Process

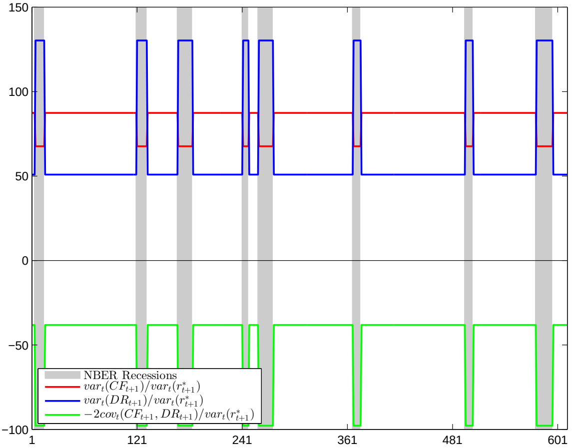
Unconditional Variance Decomposition based on Simulated Data

	Value (Ratio)
$var(CF)$	10.00 (83.54%)
$var(DR)$	8.33 (69.62%)
$-2cov(CF, DR)$	-6.36 (-53.16%)
$var(r)$	11.97 (100.00%)

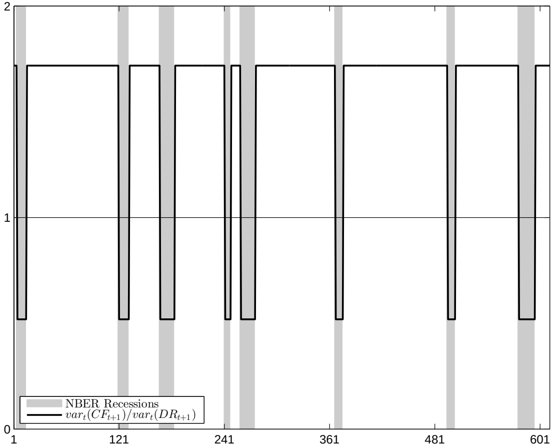
Conditional Variances based on Simulated Returns



Relative Importance based on Simulated Returns



Ratio of Conditional Variances of Cash Flows and Discount News based on Simulated Returns



Intuition - Unconditional Variance Decomposition

- Investors' risk aversion parameter is the main driving factor behind the unconditional variance of returns and its decomposition.
- The marginal rate of substitution and, thus, the stochastic discount factor depend on investors' risk aversion.
- As investors become more risk averse, the stochastic discount factor and, thus, discount rate news become more volatile.
- On the other hand, the coefficients multiplying investors' beliefs in the definition of cash flow news in this framework becomes smaller and, thus, the cash flow news becomes less volatile.
- The covariance between the two components is always positive and increases with increasing investors' risk aversion.

Intuition - Unconditional Variance Decomposition

- In the Campbell and Schiller decomposition, an increase in the variance of either discount rate or cash flow news increases the variance of returns while an increase in their covariance decreases it.
- For low levels of risk aversion, the variance of returns decreases as investors become more risk averse. This is due to the fact that the increase in the variance of discount rate news is dominated by the decreases in the variance of cash flow news and (-2 times) the covariance between the two components.
- For high levels of risk aversion, the opposite holds and the variance of returns decreases as investors become more risk averse.

Intuition - Conditional Variance Decomposition

- Transition probability matrix is the main driving factor behind the implications of this stylized model for the decomposition of the conditional variance of returns.
- Expansion periods as defined by the NBER tend to be longer than recession periods and thus also more persistent.
- Hence, investors' beliefs are more volatile in recessions than expansions.
- This in turn implies the conditional variance of returns, the conditional variance of its components and the conditional covariance between its components are higher in recessions than expansions.
- However, the variance of investors' beliefs has a bigger effect on the conditional variance of discount rate news than that of cash flow news.
- The conditional variance of discount rate news increases in recessions more than that of cash flow news and thus, its relative importance increases in recessions while that of cash flow news decreases.

Part VI

Conclusion

Conclusions

- In this paper, we analyze the decomposition of unconditional and conditional variances of returns on the S&P 500 index over the business cycle.
- To do this, we first generalize the standard return decomposition approach based on Campbell and Shiller (1988) to a framework where we model the short-run dynamics of returns and predictive variables in a Markov regime switching vector autoregressive model (MSVAR) where both the VAR parameters and residual variance matrix are assumed to switch between different values based on the underlying state of the economy.
- In contrast to the standard approach, we find that the cash flow news is more than discount rate news in determining the unconditional variance of returns based on this framework that takes into account the time-varying nature of predictive relations and variances over the business cycle.

Conclusions

- More importantly, we find that the decomposition of the conditional variance of returns depends on the underlying state of the economy.
  - The cash flow news is relatively more important than discount rate news in determining the conditional variance of returns in expansions.
  - The conditional variances of returns and its components increase in recessions.
  - The conditional variance of returns increases more than that of cash flow news and the discount rate news becomes relatively more important than cash flow news in determining the conditional variance of returns in recessions.
- Finally, we show that these results are broadly consistent with the implications of a stylized asset pricing model in which the growth rates of dividends and consumption take on different values depending on the underlying state of the economy.