Is Expected Shortfall a better risk measure than VaR?

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*The opinions expressed in this presentation are those of the author and do not necessarily reflect views of the Financial Services Authority.

Outline

Introduction (1)

Basel Committee, "Fundamental review of the trading book", May 2012:

- "A number of weaknesses have been identified with VaR, including its inability to capture **tail risk**."

- "Expected shortfall (ES) is an example of a risk metric that considers a broader range of potential outcomes than VaR."

- "Unlike VaR, ES measures the riskiness of an instrument by considering both the size and likelihood of losses above a certain threshold (eg the 99th percentile)."

- "In this way, ES accounts for tail risk in a more comprehensive manner."

Introduction (2)

Acerbi & Tasche, "Expected Shortfall: a natural coherent alternative to Value at Risk", 2002:

- "VaR is not a risk measure because it does not fulfill the axiom of **sub-additivity**."
- "This property expresses the fact that a portfolio made of sub-portfolios will risk an amount which is at most the sum of the separate amounts risked by its sub-portfolios."
- "The global risk of a portfolio will be the sum of the risks of its parts only in the case when the latter can be triggered by concurrent events, namely if the sources of these risks may conspire to act altogether."
- "In all other cases, the global risk of the portfolio will be strictly less than the sum of its partial risks thanks to risk diversification."

Introduction (3)

We will see:

- Examples of the counterintuitive behaviour of VaR.

- Nonetheless, there is not such a clear black and white picture of VaR vs. ES.

- The appropriateness of a risk measure depends on the context (e.g. time horizon, materiality of portfolio, type of risk).

- There are good reasons to believe that it is best to use both VaR and ES for risk measurement and management.

Introduction (4)

Example

- **Convention:** Losses are positive numbers, profits are negative numbers.
- Portfolio A of **100 identical but independent bonds** with face value $1mn.
- With each bond, profit/loss* of 0 with probability 0.995.
- Loss of $1mn with probability 0.005.
- Formally:

$$\mathsf{P}[X_i = 0] = 0.995,\ \mathsf{P}[X_i = 1] = 0.005, \quad i = 1, \dots, 100$$
$$X_A = \sum_{i=1}^{100} X_i$$

*Assume that interest and risk premium are paid upfront and hence are riskless.

Introduction (5)

Example continued.

- Portfolio B of **one bond** with face value \$100mn.
- Again, profit of 0 with probability 0.995.
- Loss of \$100mn with probability 0.005.
- Formally:

$$\mathsf{P}[X_B = 0] = 0.995, \quad \mathsf{P}[X_B = 100] = 0.005.$$

- **Intuition:** Portfolio A is more diversified and hence less risky than portfolio B.
- Which risk measures are aligned to this intuition?

Shortfall probability risk measures

- **Construction principle:** For a given confidence level γ (e.g. $\gamma =$ 99%), the *risk measure* $\rho(X)$ specifies a level of loss that is exceeded only with probability less than $1 - \gamma$.

- *Loss* is understood as loss relative to expected profit and loss, hence Loss $= X - \mathrm{E}[X]$.

- Formally, $\rho(X)$ should satisfy

$$\mathrm{P}[X - \mathrm{E}[X] > \rho(X)] \;\leq\; 1 - \gamma. \tag{6.1}$$

- Examples: (Scaled) standard deviation, Value-at-Risk (VaR), Expected Shortfall (ES).

- Alternative construction principles (examples): Measure of volatility, shortfall as loss beyond fix limit.

Scaled Standard Deviation (1)

- Scaled **standard deviation as risk measure:**

$$\sigma_a(X) \;=\; a\sqrt{\mathrm{var}[X]} \;=\; a\sqrt{\mathrm{E}\Big[(X-\mathrm{E}[X])^2\Big]}. \tag{7.1}$$

- By Chebychev's inequality:

$$\begin{aligned} \mathrm{P}[X-\mathrm{E}[X] > \sigma_a(X)] \;&=\; \mathrm{P}\Big[X-\mathrm{E}[X] > a\sqrt{\mathrm{var}[X]}\Big] \\ &\le\; \mathrm{P}\Big[|X-\mathrm{E}[X]| \ge a\sqrt{\mathrm{var}[X]}\Big] \\ &\le\; a^{-2}. \end{aligned} \tag{7.2}$$

- Hence, choosing $a = \frac{1}{\sqrt{1-\gamma}}$ provides $\mathrm{P}[X-\mathrm{E}[X] > \sigma_a(X)] \;\le\; 1-\gamma$. For instance, $\gamma = 0.99$ implies $a = 10$.

- Alternatively: Choosing a such that (6.1) holds for, e.g., normally distributed X. Then, $\gamma = 0.99$ implies $a = \Phi^{-1}(0.99) \approx 2.33$.

- $\sigma_a(X)$ is **sub-additive.**

Scaled Standard Deviation (2)

Example (slides 4 and 5)

- X_A is binomially distributed with success probability 0.005 and size parameter 100. Hence

$$\mathsf{E}[X_A] = 100 \cdot 0.005 = 0.5,$$
$$\mathsf{var}[X_A] = 100 \cdot 0.005 \cdot 0.995 = 0.4975.$$

- $X_B = 100\,Z$ where Z is Bernoulli distributed with success probability 0.005. Hence

$$\mathsf{E}[X_B] = 100 \cdot 0.005 = 0.5,$$
$$\mathsf{var}[X_B] = 100^2 \cdot 0.005 \cdot 0.995 = 49.75.$$

- $\sigma_{10}(X_A) = 7.05,\qquad \sigma_{10}(X_B) = 70.53,$
  $\sigma_{2.33}(X_A) = 1.64,\qquad \sigma_{2.33}(X_B) = 16.41$

Scaled Standard Deviation (3)

Example continued (slides 4 and 5)

- σ_a is aligned to intuition:

$$\sigma_{10}(X_A) < \sigma_{10}(X_B), \qquad \sigma_{2.33}(X_A) < \sigma_{2.33}(X_B).$$

- With a chosen according to (7.2), the **shortfall probability is correctly respected**:

$$\begin{aligned} \mathsf{P}[X_A > \sigma_{10}(X_A) + \mathsf{E}[X_A]] &= \mathsf{P}[X_A > 7.55] \\ &= 4.8\text{e-}08 < 1 - 0.99, && (9.1) \\ \mathsf{P}[X_B > \sigma_{10}(X_B) + \mathsf{E}[X_B]] &= \mathsf{P}[Z > \sigma_{10}(Z) + \mathsf{E}[Z]] && (9.2) \\ &= \mathsf{P}[Z > 0.71] = 0.005 < 1 - 0.99. \end{aligned}$$

Scaled Standard Deviation (4)

Example continued (slides 4 and 5)

- With a chosen with regard to normally distributed losses, the **shortfall probability is not always respected**:

$$\begin{aligned}
\mathsf{P}[X_A > \sigma_{2.33}(X_A) + \mathsf{E}[X_A]] &= \mathsf{P}[X_A > 2.14] \\
&= 0.0141 > 1 - 0.99, \\
\mathsf{P}[X_B > \sigma_{2.33}(X_B) + \mathsf{E}[X_B]] &= \mathsf{P}[Z > \sigma_{2.33}(Z) + \mathsf{E}[Z]] \\
&= \mathsf{P}[Z > 0.17] = 0.005 < 1 - 0.99.
\end{aligned}$$

- **Conclusion:** σ_a can be a meaningful measure of tail risk and concentration. However, (9.1) shows that it may 'overshoot' significantly – too much conservatism.

Scaled Standard Deviation (5)

- Comparing values of a in (7.1) according to Chebychev, standard normal, and standardised Student t* (3 degrees of freedom) approaches.

Level γ	0.9	0.95	0.99	0.995	0.999	0.9995
Chebychev	3.16	4.47	10.00	14.14	31.62	44.72
Normal	1.28	1.64	2.33	2.58	3.09	3.29
Student (df=3)	0.95	1.36	2.62	3.37	5.90	7.46

- Chebychev approach guarantees that shortfall probability is respected for every distribution with finite variance.
- Even compared to heavy-tail distribution, Chebychev approach yields very conservative values.

*If X is Student-t-distributed with degrees of freedom $n \geq 3$, then $\mathrm{var}[X] = \frac{n}{n-2}$.

Value-at-Risk (1)

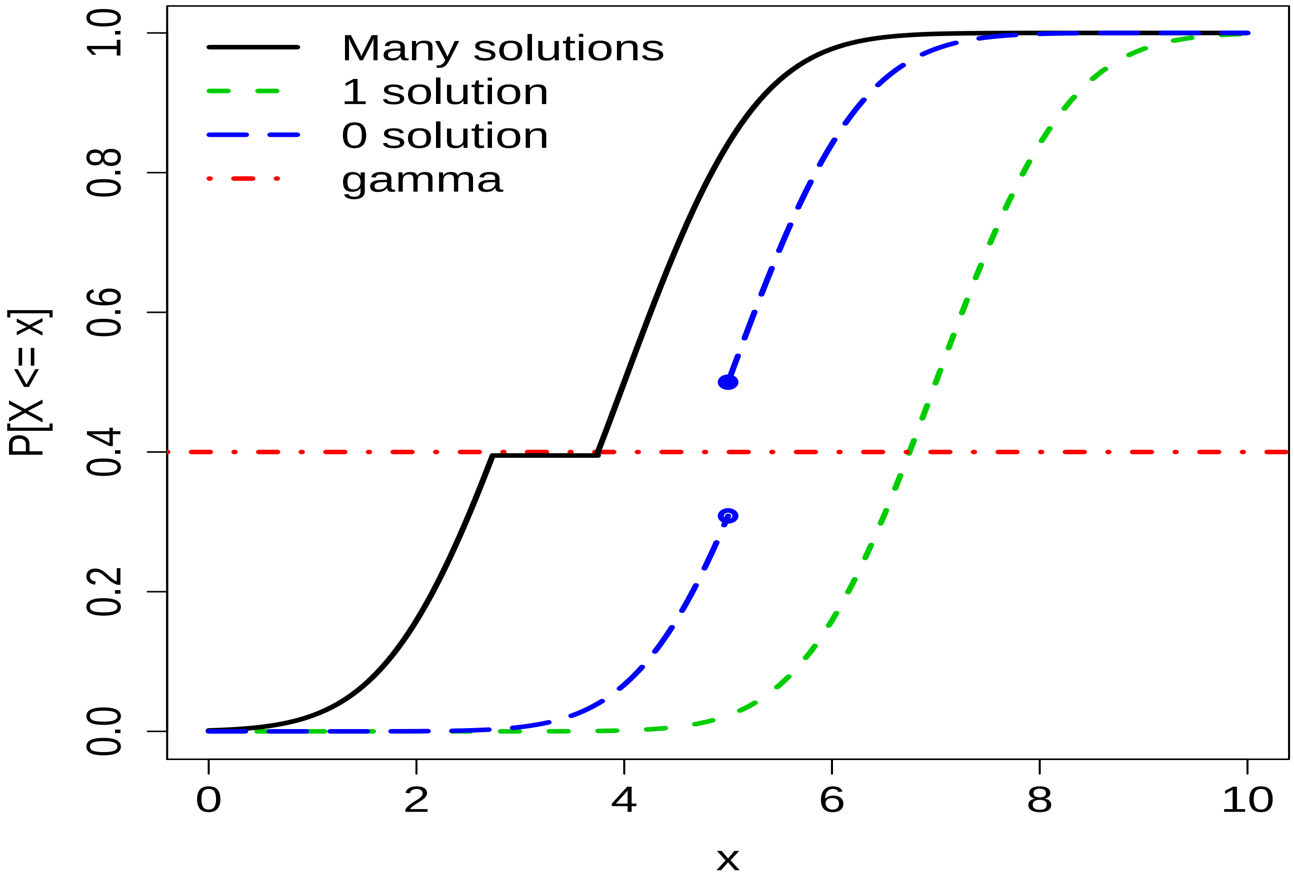
- For $\gamma \in (0,1)$: γ-quantile $q_\gamma(X) = \min\{x : \mathsf{P}[X \le x] \ge \gamma\}$.
- In finance, $q_\gamma(X)$ is called **Value-at-Risk** (VaR).
- If X has a positive density $f(x)$ (i.e. $\mathsf{P}[X \le x] = \int\limits_{-\infty}^{x} f(t)\,dt$), then $q_\gamma(X)$ is the unique solution of $\mathsf{P}[X \le x] = \gamma$.
- Example: **Quantile / VaR-based** risk measure.

$$\rho_{\mathsf{VaR},\gamma}(X) \;=\; q_\gamma(X) - \mathsf{E}[X]. \tag{12.1}$$

- By definition, we have $\mathsf{P}[X > \rho_{\mathsf{VaR},\gamma}(X) + \mathsf{E}[X]] \le 1 - \gamma$.

Value-at-Risk (2)

Solving Equation $\mathrm{P}[X \leq x] = \gamma$



Value-at-Risk (3)

Example (slides 4 and 5)

- Portfolio A (100 independent bonds with probability of default 0.005):

$$q_{0.99}(X_A) = 3 \;\Rightarrow\; \rho_{\mathrm{VaR},0.99}(X_A) = 3 - 0.5 = 2.5.$$

- Portfolio B (1 bond with probability of default 0.005):

$$q_{0.99}(Z) = 0 \;\Rightarrow\; \rho_{\mathrm{VaR},0.99}(X_B) = 100 \cdot 0 - 0.5 = -0.5.$$

- Counterintuitive – and **violation of subadditivity** because

$$\begin{aligned}\rho_{\mathrm{VaR},0.99}(X_A) = 2.5 &> \sum_{i=1}^{100} \rho_{\mathrm{VaR},0.99}(X_i) \\ &= \sum_{i=1}^{100} (q_{0.99}(Z) - 0.005) = -0.5\end{aligned}$$

Expected Shortfall (1)

- VaR as risk measure is criticized* because it is, in general, not a **sub-additive** risk measure, i.e.

$$\rho_{\mathsf{VaR},\gamma}(X^* + X^{**}) \;\not\leq\; \rho_{\mathsf{VaR},\gamma}(X^*) + \rho_{\mathsf{VaR},\gamma}(X^{**})$$

  can occur.

- Proposed remedy **Expected Shortfall** (ES):

$$\begin{aligned} \mathsf{ES}_\gamma(X) &= \tfrac{1}{1-\gamma}\int_\gamma^1 q_u(X)\,du \\ &= \mathsf{E}[X \mid X \geq q_\gamma(X)] \\ &\quad + \Big(\mathsf{E}[X \mid X \geq q_\gamma(X)] - q_\gamma(X)\Big)\left(\tfrac{\mathsf{P}[X \geq q_\gamma(X)]}{1-\gamma} - 1\right). \end{aligned} \tag{15.1}$$

*See, in particular, Artzner, P., Delbaen, F., Eber, J.-M. and D. Heath: Coherent measures of risk. Mathematical Finance 9, 1999, 203-228.

Expected Shortfall (2)

- If $\mathsf{P}[X = q_\gamma(X)] = 0$ (in particular, if X has a density),

$$\mathsf{ES}_\gamma(X) \;=\; \mathsf{E}[X \mid X \geq q_\gamma(X)].$$

- In general:

$$\mathsf{E}[X \mid X \geq q_\gamma(X)] \;\leq\; \mathsf{ES}_\gamma(X) \;\leq\; \mathsf{E}[X \mid X > q_\gamma(X)] \qquad (16.1)$$

- **ES-based risk measure:**

$$\rho_{\mathsf{ES},\gamma}(X) \;=\; \mathsf{ES}_\gamma(X) - \mathsf{E}[X]. \qquad (16.2)$$

- ES is sub-additive (non-trivial proof).

- $u \mapsto q_u(X)$ is non-decreasing $\Rightarrow$

$$\rho_{\mathsf{ES},\gamma}(X) \;=\; \tfrac{1}{1-\gamma}\int_\gamma^1 q_u(X)\,du - \mathsf{E}[X] \geq \; \rho_{\mathsf{VaR},\gamma}(X) \qquad (16.3)$$

Expected Shortfall (3)

Example (slides 4 and 5)

- Portfolio A (100 independent bonds with probability of default 0.005): $q_{0.99}(X_A) = 3 \;\Rightarrow$

$$\begin{aligned} \mathsf{P}[X_A \geq q_{0.99}(X_A)] &= \mathsf{P}[X_A \geq 3] = 0.0141 \\ \mathsf{E}[X_A \,|\, X_A \geq q_{0.99}(X_A)] &= \mathsf{E}[X_A \,|\, X_A \geq 3] = 3.13 \\ \mathsf{E}[X_A \,|\, X_A > q_{0.99}(X_A)] &= \mathsf{E}[X_A \,|\, X_A > 3] = 4.103 \\ \mathsf{ES}_{0.99}(X_A) &= 3.18 \\ \rho_{\mathsf{ES},0.99}(X_A) &= \mathsf{ES}_{0.99}(X_A) - \mathsf{E}[X_A] = 2.68 \end{aligned}$$

- Note that $\mathsf{P}[X_A \geq q_{0.99}(X_A)]$ is not much greater than $1-\gamma = 0.01$. By (15.1), this implies that $\mathsf{ES}_{0.99}(X_A)$ is close to $\mathsf{E}[X_A \,|\, X_A \geq q_{0.99}(X_A)]$.

Expected Shortfall (4)

Example (slides 4 and 5)

- Portfolio B (1 bond with probability of default 0.005):
  $q_{0.99}(Z) = 0 \;\Rightarrow$

$$\begin{aligned} \mathsf{P}[X_B \geq q_{0.99}(X_B)] &= \mathsf{P}[Z \geq 0] = 1 \\ \mathsf{E}[X_B \,|\, X_B \geq q_{0.99}(X_B)] &= 100 \cdot \mathsf{E}[Z] = 0.5 \\ \mathsf{E}[X_B \,|\, X_B > q_{0.99}(X_B)] &= 100 \cdot \mathsf{E}[Z \,|\, Z > 0] = 100 \\ \mathsf{ES}_{0.99}(X_B) &= 50 \\ \rho_{\mathsf{ES},0.99}(X_B) &= \mathsf{ES}_{0.99}(X_B) - \mathsf{E}[X_B] = 49.5. \end{aligned}$$

- $\mathsf{P}[X_B \geq q_{0.99}(X_B)] \gg 1 - \gamma = 0.01$ implies
  $\mathsf{ES}_{0.99}(X_B) \gg \mathsf{E}[X_B \,|\, X_B \geq q_{0.99}(X_B)]$.

- Sub-additivity holds:

$$\mathsf{ES}_{0.99}(X_A) = 3.18 < \sum_{i=1}^{100} \mathsf{ES}_{0.99}(X_i) = \mathsf{ES}_{0.99}(X_B) = 50$$

Expected Shortfall (5)

Back-testing of ES

- Back-testing of ES can be based on the representation of ES as integrated VaR:

$$
\begin{aligned}
\mathrm{ES}_\gamma(X) &= \frac{1}{1-\gamma}\int_\gamma^1 q_u(X)\,du \\
&\approx 1/4\,\Big(q_\gamma(X) + q_{0.75\,\gamma+0.25}(X) \\
&\qquad + q_{0.5\,\gamma+0.5}(X) + q_{0.25\,\gamma+0.75}(X)\Big)
\end{aligned}
\tag{19.1}
$$

- Hence, if $q_\gamma(X)$, $q_{0.75\,\gamma+0.25}(X)$, $q_{0.5\,\gamma+0.5}(X)$, and $q_{0.25\,\gamma+0.75}(X)$ are shown to be correct then $\mathrm{ES}_\gamma(X)$ is correct, too.

- Drawback: For same level of certainty a much longer sample is needed for $\mathrm{ES}_\gamma(X)$ than for $\mathrm{VaR}_\gamma(X)$.

VaR vs. ES (1)

VaR and ES for different distributions (standardised)

	Normal	Exponential	Weibull	Lognorm	t3
VaR 90%	1.282	1.303	0.738	1.161	0.946
ES 90%	1.755	2.303	2.215	2.268	1.681
VaR 99%	2.326	3.605	4.295	3.750	2.622
ES 99%	2.665	4.605	6.802	5.250	4.043
VaR 99.9%	3.090	5.908	10.223	7.259	5.897
ES 99.9%	3.367	6.908	13.759	9.254	8.897
VaR 99.99%	3.719	8.210	18.521	11.923	12.819
ES 99.99%	3.958	9.210	23.088	14.524	19.252
VaR 99.999%	4.265	10.513	29.191	17.998	27.671
ES 99.999%	4.479	11.513	34.787	21.329	41.517

VaR vs. ES (2)

Ratios of VaR and ES for different levels and distributions

	Normal	Exponential	Weibull	Lognorm	t3
ES/VaR 90%	1.369	1.768	3.000	1.954	1.777
ES/VaR 99%	1.146	1.277	1.584	1.400	1.542
ES/VaR 99.9%	1.090	1.169	1.346	1.275	1.509
ES/VaR 99.99%	1.064	1.122	1.247	1.218	1.502
ES/VaR 99.999%	1.050	1.095	1.192	1.185	1.500

- Normal and exponential: Exponential or faster decay of tail probabilities
- Weibull and Lognormal: Sub-exponential but faster than polynomial decay
- t3 (Student with 3 degrees of freedom): Polynomial decay

VaR vs. ES (3)

Observations on VaR and ES

- Even with identical first 2 moments magnitudes of VaR and ES depend strongly on type of loss distribution

- Distributions with similar VaR and/or ES for moderate confidence levels may have very different VaR and/or ES for high levels

- The relative difference between VaR and ES becomes smaller for higher confidence levels

- A high relative difference between VaR and ES may indicate tail behaviour which is heavier than normal

- Calculating VaR and ES at different confidence levels can help to identify fat loss tails

Measuring diversification (1)

- Sources of concentration risk in portfolios:
  - Large single risks
  - Groups of highly correlated small- and medium-sized risks
- **Worst case: deterministic dependence**, i.e. if one position sees a big loss then also all other positions make big losses (extreme concentration).
- In practice, dependence will be weaker in most cases.
- Worst case should be judged most risky, i.e. should require highest capital reserve.

Measuring diversification (2)

- Mathematical description of deterministic dependence: **Co-monotonicity.**

- Random variables X and Y are **co-monotonic** if they can be represented as increasing functions of a common factor Z, i.e.

$$X = h_X(Z) \quad \text{and} \quad Y = h_Y(Z)$$

  with h_X, h_Y both non-decreasing or both non-increasing.

- Co-monotonicity generalizes the concept of linear dependence:

$$\text{corr}[X, Y] = 1 \quad \Rightarrow \quad X, Y \text{ co-monotonic}$$

Measuring diversification (3)

- X, Y co-monotonic

$$\Rightarrow \quad \begin{aligned} \text{VaR}_\gamma(X+Y) &= \text{VaR}_\gamma(X) + \text{VaR}_\gamma(Y) \\ \text{ES}_\gamma(X+Y) &= \text{ES}_\gamma(X) + \text{ES}_\gamma(Y) \end{aligned}$$

- Standard deviation:

$$X, Y \text{ co-monotonic } \not\Rightarrow \sigma_a(X+Y) = \sigma_a(X) + \sigma_a(Y)$$

- ES and standard deviation are sub-additive, VaR is not sub-additive:

$$\sigma_a(X+Y) \le \sigma_a(X) + \sigma_a(Y)$$

$$\text{ES}_\gamma(X+Y) \le \text{ES}_\gamma(X) + \text{ES}_\gamma(Y)$$

$$\text{VaR}_\gamma(X+Y) \not\le \text{VaR}_\gamma(X) + \text{VaR}_\gamma(Y)$$

Measuring diversification (4)

- If the capital reserve is measured with ES, **co-monotonicity represents worst case scenarios.**

- Does not necessarily hold for VaR*. Even in case of independent loss variables, portfolio-wide VaR can exceed the sum of stand-alone VaRs.

- For standard deviation, not co-monotonicity but full correlation yields worst case scenarios.

*See, e.g., Embrechts, Höing & Puccetti: Worst VaR Scenarios (2005)

Measuring diversification (5)

- Idea: compare capital reserve for portfolio to capital reserve for worst case portfolio.

- Worst case by assuming co-monotonicity for all profit/loss variables.

- X portfolio profit/loss, X_i loss with i-th risk, i.e. $X = \sum_{i=1}^{m} X_i$.

- Capital reserve given by measure ρ of Unexpected Loss, e.g.

$$\rho(X) = \mathrm{VaR}_\gamma(X) - \mathrm{E}[X] \quad \text{or}$$
$$\rho(X) = \mathrm{ES}_\gamma(X) - \mathrm{E}[X]$$

- **Diversification index** for portfolio:

$$\mathrm{DI}_\rho(X) = \frac{\rho(X)}{\sum_{i=1}^{m} \rho(X_i)} \tag{27.1}$$

Measuring diversification (6)

- ρ sub-additive $\quad\Rightarrow\quad \mathrm{DI}_\rho(X) \le 1$

- ρ additive for co-monotonic profit/loss variables
$\Rightarrow \quad \mathrm{DI}_\rho(X) = 1$ for worst case scenario.

- $\mathrm{DI}_\rho(X)$ close to one $\quad\Leftrightarrow\quad$ Portfolio not well diversified

- Need to define indices $\mathrm{DI}_\rho(X_i \,|\, X)$ at risky position level in order to identify drivers of concentration risk.

- **Desirable:** $\mathrm{DI}_\rho(X_i \,|\, X) > \mathrm{DI}_\rho(X) \Rightarrow$
Reducing i-th position "improves" portfolio.

Measuring diversification (7)

- ρ positively homogeneous $\Leftrightarrow \rho(h\,X) = h\,\rho(X), h > 0$

- **Euler-decomposition** of capital reserve:
  $\rho(X)$ positively homogeneous $\Rightarrow$

$$\rho(X) = \sum_{i=1}^{m} \rho(X_i \,|\, X),$$

$$\rho(X_i \,|\, X) = \frac{d}{dh}\rho(h\,X_i + X)\Big|_{h=0}$$

- Examples on Slide 30.

- **Diversification index** for single position (or sub-portfolio):

$$\mathrm{DI}_\rho(X_i \,|\, X) \;=\; \frac{\rho(X_i \,|\, X)}{\rho(X_i)} \tag{29.1}$$

Measuring diversification (8)

- **Derivative of standard deviation:**

$$\sigma_a(X_i\,|\,X) \;=\; a\,\frac{\mathsf{cov}(X_i, X)}{\sqrt{\mathsf{var}(X)}}.$$

- **Derivative of Value-at-Risk:**

$$\rho_{\mathsf{VaR},\gamma}(X_i\,|\,X) \;=\; \mathsf{E}\Big[X_i \mid X = q_\alpha(X)\Big] - \mathsf{E}[X_i].$$

- **Derivative of Expected Shortfall:** (continuous case)

$$\rho_{\mathsf{ES},\gamma}(X_i\,|\,X) \;=\; \mathsf{E}\Big[X_i\,|\,X \geq q_\alpha(X)\Big] - \mathsf{E}[X_i].$$

Measuring diversification (9)

- $\rho(X)$ positively homogeneous $\Rightarrow$

$$\rho(X_i \,|\, X) \le \rho(X_i) \quad \Leftrightarrow \quad \rho \text{ sub-additive}$$

- ρ sub-additive $\quad \Rightarrow \quad \mathrm{DI}_\rho(X_i \,|\, X) \le 1$

- **Portfolio "best" diversified** (i.e. $\mathrm{DI}_\rho(X)$ minimal)

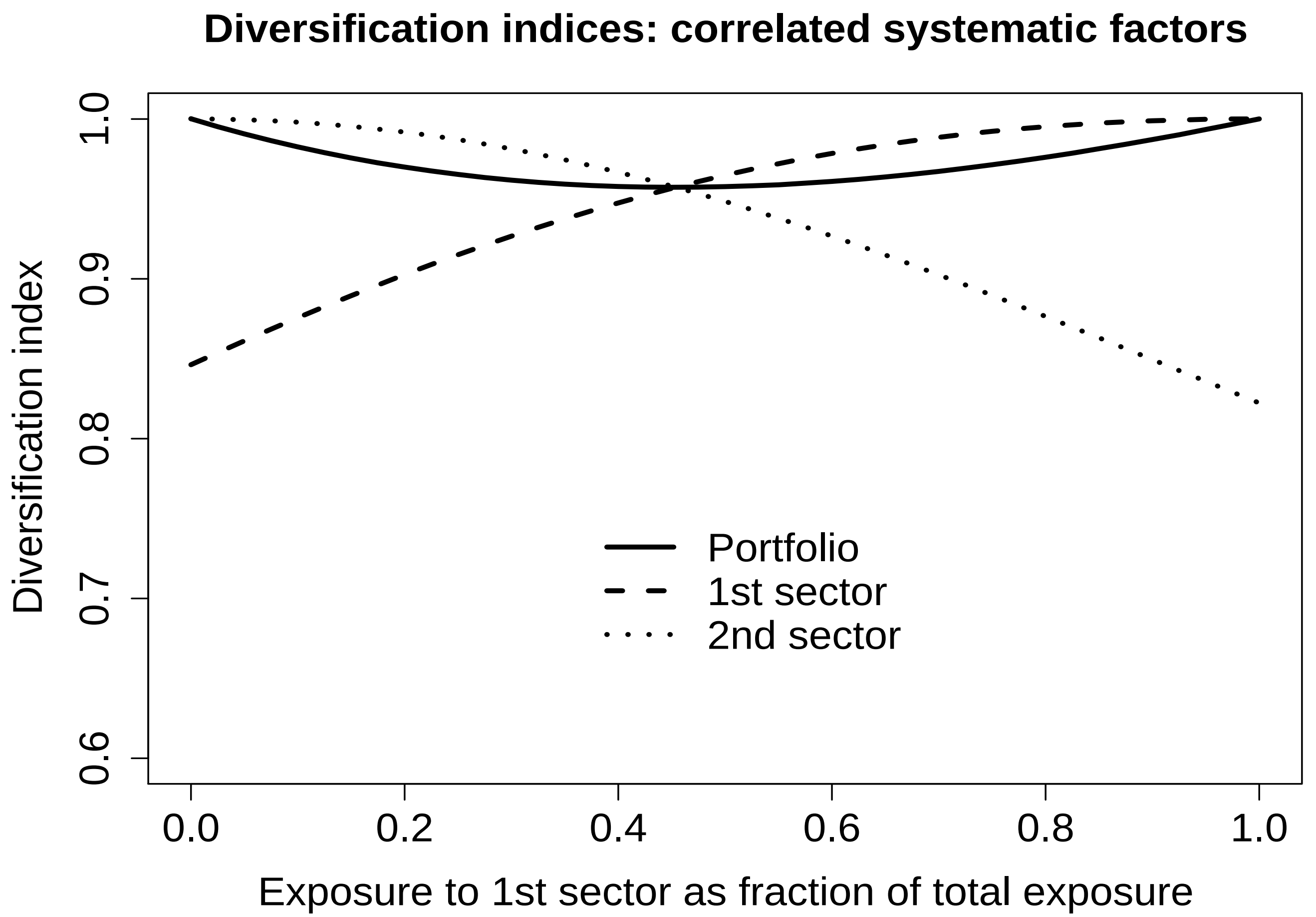
$$\Rightarrow \quad \mathrm{DI}_\rho(X_i \,|\, X) = \mathrm{DI}_\rho(X) \text{ for all } i \tag{31.1}$$

- It can be proved that definition of $\mathrm{DI}_\rho(X_i \,|\, X)$ with (31.1) is unique.

- $\max_{i=1,\ldots,m} |\mathrm{DI}_\rho(X_i \,|\, X) - \mathrm{DI}_\rho(X)|$ is indicator of lacking diversification.

Measuring diversification (10)

Diversification indices of portfolio and of sub-portfolios A and B. Represented as function of weight of sub-portfolio A.



Conclusions

- As shown by example, VaR has deficiencies with regard to capturing tail risk and measuring diversification.

- Standard deviation (even if scaled) is no realistic alternative.

- Expected shortfall is a meaningful alternative.

- Measuring both VaR and ES can help detecting heavy tail loss distributions.

- Back-testing of ES requires longer time series than back-testing of VaR.