SHARING THE LONGEVITY RISK BETWEEN ANNUITANTS AND ANNUITY PROVIDER *

ANNAMARIA OLIVIERI
University of Parma (Italy)
annamaria.olivieri@unipr.it

Meeting of the Working Group on Risk – ESSEC Business School

Paris, 18 February 2013

* Based on joint work with Ermanno Pitacco, University of Trieste (Italy)

Motivation

Traditional life annuities imply financial and longevity guarantees

Longevity guarantees

Basic (and essential, so far) feature of the life annuity:
pooling $\Rightarrow$ longevity guarantee
On the other hand: pooling $\Rightarrow$ unavailability of money for bequest
$\Rightarrow$ life annuities not attractive in the early years after retirement

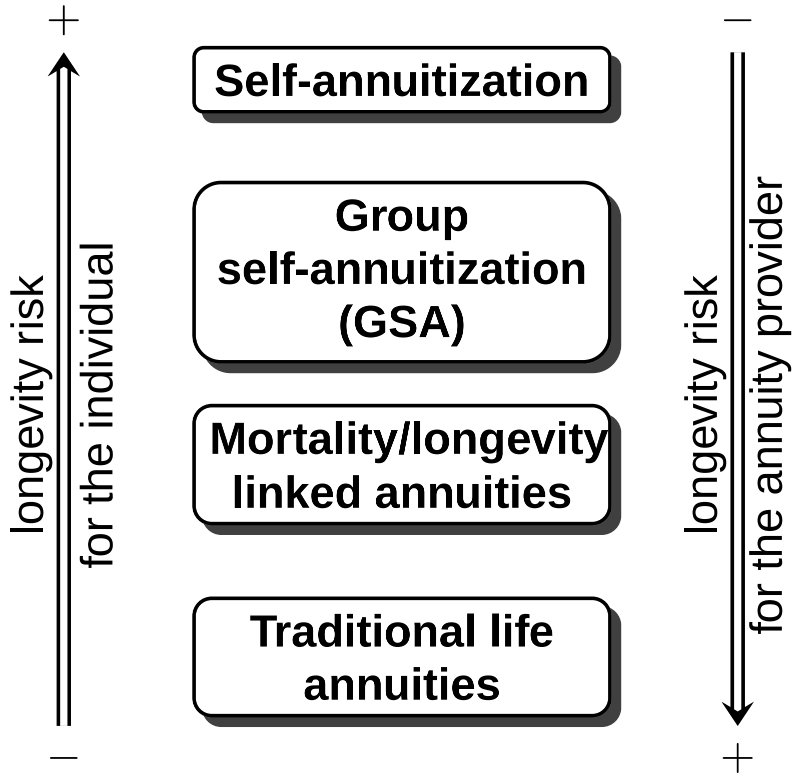
The value of longevity guarantees has increased substantially in the latest decades, because of major unanticipated mortality improvements ($+$ reduction of interest rates)

Their pricing requires stochastic models not yet fully reliable or not yet easily tractable

Further: if applied, currently the fees for these guarantees would be high, and make the product even less attractive than how it is already perceived

Motivation (*cont'd*)

Arranging the post-retirement income:
Alternative solutions, implying a different sharing of the longevity risk



We focus on mortality/longevity linked annuities:
the annuity benefit is adjusted either to

- The mortality experienced by a specific pool of pensioners/annuitants, or
- The mortality experienced by the general population, or
- New projected life tables

Literature:

- Lüty et al. [2001]: adjustment to updated mortality forecasts
- Richter and Weber [2011]: link to actual mortality experience
- van de Ven and Weale [2008]: adjustment so to transfer the aggregate longevity risk
- Denuit et al. [2011]: adjustment to the unexpected longevity improvements experienced by a given reference population

Our aim:

1. To investigate the features, and check the feasibility, of alternative arrangements in terms of benefit trajectories
2. To assess the value of practicable solutions, in particular from the point of view of the annuity provider
3. To quantify the longevity risk charged to individuals and the annuity provider

In this presentation: Step 1

Several possible linking solutions to mortality/longevity are considered, and compared

We admit participation to extra-returns on investments

Linking arrangements

Longevity risk

- Idiosyncratic (process or insurance risk): some individuals live longer than others $\Rightarrow$ temporary and subject to pooling effects
- Aggregate: on average, people live longer than expected $\Rightarrow$ permanent, no pooling effect

General purpose of linking arrangements: sharing losses (or profits) originated by aggregate longevity risk, so that

1. Annual payouts are close to a target amount $\Rightarrow$ model calibration based on more direct measures of longevity
2. The portfolio reserve is close to the available assets $\Rightarrow$ more suitable for a transfer of the aggregate longevity risk only

Smoothing effects, so to avoid to share with the individuals temporary profits/losses (namely, due to the idiosyncratic risk or financial returns) $\Rightarrow$ adjustments every k years (say, $k = 3, 5$) & multiperiod (namely, k-years) longevity measures

Basic items in defining the linking rule:

1. Choice of a mortality dataset
2. Choice of the adjustment coefficient of the benefit

Examples of mortality datasets

- Actual number of surviving annuitants:

$$N_{x+1}, N_{x+2}, \ldots$$

- Actual number of survivors in a reference cohort:

$$L_{x+1}, L_{x+2}, \ldots$$

- Expected number of surviving annuitants, according to (the initial) information $\mathcal{F}_0$ (for example: $\mathcal{F}_0 =$ life table at time 0):

$$\mathbb{E}[N_{x+1}|\mathcal{F}_0], \mathbb{E}[N_{x+2}|\mathcal{F}_0], \ldots$$

- Expected number of survivors in the reference cohort, according to (the initial) information $\mathcal{F}_0$:

$$\mathbb{E}[L_{x+1}|\mathcal{F}_0], \mathbb{E}[L_{x+2}|\mathcal{F}_0], \dots$$

- Expected number of surviving annuitants, according to (the current) updated information $\mathcal{F}'_{0,t}$:

$$\mathbb{E}[N_{x+t}|\mathcal{F}'_{0,t}], \mathbb{E}[N_{x+t+1}|\mathcal{F}'_{0,t}], \dots$$

  for example: $\mathcal{F}'_{0,t} = \{\mathcal{F}_0; N_{x+1}, \dots, N_{x+t-1}\}$

- Expected number of survivors in the reference cohort, according to (the current) new information $\mathcal{F}_t$:

$$\mathbb{E}[L_{x+t}|\mathcal{F}_t], \mathbb{E}[L_{x+t+1}|\mathcal{F}_t], \dots$$

  for example: $\mathcal{F}_t =$ new projected life table at time t

Definition of the adjustment coefficients

Various approaches possible. In particular, the definition can be:

- *retrospective*: directly involving observed mortality, in terms of $N_{x+1}, N_{x+2}, \dots$ (*indemnity-based*), or $L_{x+1}, L_{x+2}, \dots$ (*index-based*)
- *prospective*: relying on updated mortality forecasts, e.g. $\mathbb{E}[L_{x+t}|\mathcal{F}_t], \mathbb{E}[L_{x+t+1}|\mathcal{F}_t], \dots$

Longevity measures based on:

- Number of survivors
- Actuarial quantities, such as portfolio reserve and assets, actuarial value of an annuity

Mortality/longevity adjustment "Group Self-Annuitization" (GSA)-like:

every k years: $$\underbrace{b_t}_{\text{individual benefit at time } t} = b_{t-1} \times \underbrace{\frac{\overbrace{A_t}^{\text{assets}}}{\underbrace{V_t^{[\mathrm{P}]}}_{\text{portfolio reserve}}}}_{\text{coefficient of adjustment, } \gamma_t}$$

If $A_t \lesseqgtr V_t^{[\mathrm{P}]}$ because of financial returns, or mortality experience, or issue of a new life table

$$\Rightarrow \gamma_t \lesseqgtr 1 \Rightarrow V_{t^+}^{[\mathrm{P}]} = A_t$$

Apart for intermediate times, all the risks are borne by the individuals, as a pool

Linking arrangements (*cont'd*)

Mortality/longevity adjustment based on the number of survivors:

life table set at time 0

survival rate in life table $\mathcal{F}_0$

every k years:

$$b_t = b_{t-1} \times \underbrace{\frac{\mathbb{E}[N_{x+t}|\mathcal{F}_0]}{N_{x+t}}}_{m_t^{[1]}} = b_{t-1} \times \frac{{}_tp_x^{\mathcal{F}_0}}{{}_t\tilde{p}_x}$$

annuitants, out of initial N_x

realized survival rate

If $N_{x+t} \gtrless \mathbb{E}[N_{x+t}|\mathcal{F}_0]$ (i.e.: if ${}_t\tilde{p}_x \gtrless {}_tp_x^{\mathcal{F}_0}$) $\Rightarrow m_t^{[1]} \lessgtr 1$

- annuitants bear a part of the aggregate longevity risk (but possibly also some idiosyncratic longevity risk)
- life table risk with the provider
- not necessarily $V_{t^+}^{[\mathrm{P}]} = A_t$, but through the adjustment the difference should be reduced

Variant:

current life table

$$m_t^{[2]} = \frac{\mathbb{E}[N_{x+t}|\mathcal{F}_t]}{N_{x+t}}$$

Variant:

life table at time 0, updated to the mortality experienced in the portfolio in the time-interval $(0,t)$

$$m_t^{[3]} = \frac{\mathbb{E}[N_{x+t}|\mathcal{F}'_{0,t}]}{N_{x+t}}$$

Variant 2 (potentially) implies more longevity risk for the individuals, variant 3 possibly less

Linking arrangements (*cont'd*)

The coefficients $m_t^{[1]}$, $m_t^{[2]}$, $m_t^{[3]}$ are *indemnity-based*, as they compare the expected survival rate with the survival rate realized in the pool

Variants: *index-based* adjustment coefficients

$$m_t^{[4]} = \frac{\mathbb{E}[L_{x+t}|\mathcal{F}_0]}{L_{x+t}} = b_{t-1} \times \frac{{}_tp_x^{\mathcal{F}_0}}{{}_t\tilde{p}_x}$$

survivors, out of initial L_x, in a reference population

survival rate realized in the reference population

$$m_t^{[5]} = \frac{\mathbb{E}[L_{x+t}|\mathcal{F}_t]}{L_{x+t}}$$

In principle, less idiosyncratic longevity risk is transferred to individuals

Basis risk borne by the annuity provider

Further variants:
Ratios based on the survival rates of different life tables

$$m_t^{[6]} = \frac{\mathbb{E}[N_{x+t}|\mathcal{F}_0]}{\mathbb{E}[N_{x+t}|\mathcal{F}_t]} = \frac{{}_tp_x^{\mathcal{F}_0}}{{}_tp_x^{\mathcal{F}_t}}$$

$$m_t^{[7]} = \frac{\mathbb{E}[N_{x+t}|\mathcal{F}_0]}{\mathbb{E}[N_{x+t}|\mathcal{F}'_{0,t}]}$$

$$m_t^{[8]} = \frac{\mathbb{E}[N_{x+t}|\mathcal{F}'_{0,t}]}{\mathbb{E}[N_{x+t}|\mathcal{F}_t]}$$

Idiosyncratic and part of the aggregate risk retained by the provider

Mortality/longevity adjustment "unfunded liabilities"-like:

assets accumulated at the return credited to policyholders, according to the financial participation arrangement

every k years: $$b_t = b_{t-1} \times \underbrace{\frac{A_t^f}{V_t^{[\mathrm{P}]}}}_{m_t^{[9]}}$$

Because of financial guarantees, it may result $A_t^f \neq A_t$

However, since $V_{t^+}^{[\mathrm{P}]} = A_t^f$, then $V_{t^+}^{[\mathrm{P}]}$ should be close to A_t

The risk transferred to individuals is mainly the aggregate longevity risk

Mortality/longevity adjustment based on the actuarial value of the life annuity:

every k years: $$b_t = b_{t-1} \times \underbrace{\frac{\mathbb{E}[a_{\lceil K_{x+t} \rceil} | \mathcal{F}_0]}{\mathbb{E}[a_{\lceil K_{x+t} \rceil} | \mathcal{F}_t]}}_{m_t^{[10]}}$$

The aggregate longevity risk connected to the possible update of the life table is transferred to individuals

Consequences for the provider:

- individual reserve: if, for example, $m_t^{[10]} < 1$, then: $V_{t^+} = b_t \, \mathbb{E}[a_{\lceil K_{x+t} \rceil} | \mathcal{F}_t] < V_t = b_{t-1} \, \mathbb{E}[a_{\lceil K_{x+t} \rceil} | \mathcal{F}_t]$
- however, the portfolio reserve $V_{t^+}^{[\mathrm{P}]} = N_{x+t} \, V_{t^+}$ retains the uncertainty on the number of survivors
- thus it may turn out $V_{t^+}^{[\mathrm{P}]} \neq A_t$

Matching the mortality/longevity linking to a
participation to the extra-return on investments:

Every year

$$b_t = b_{t-1} \times \underbrace{\max\left\{\frac{1+\eta_t\, g_t}{1+i'}, 1\right\}}_{f_t^{[1]}}$$

$\Rightarrow$ interest rate i' guaranteed year by year
(g_t: realized yield; η_t: participation rate;
$\max\{\eta_t\, g_t, i'\}$: yield credited to policyholders)

Every k years

$$b_t = b_{t-1} \times f_t^{[1]} \times m_t^{[\cdot]}$$

Variant: $f_t^{[2]}$ such that the (annual) interest rate i' is guaranteed on average every k years

Benefit trajectories

In all cases:

Entry age $x = 65$

Mortality/longevity adjustments every $k = 5$ years

Maximum age for mortality/longevity adjustment (apart from the GSA): 95 (i.e., time 30)

One cohort

$\frac{A_{\omega-x}}{A_0}$: remaining assets at cohort's exhaustion, as a percentage of the initial assets ($=$ premiums)

Traditional pricing: $A_0 = N_x \times \mathbb{E}[a_{\overline{K_x}|}|\mathcal{F}_0]$

Experienced mortality: 90% of the best-estimate (as at time 0, i.e. $\mathcal{F}_0$)

Case a:

- Best-estimate life table unchanged
- No extra-return on investments

Case b:

- New projected life table at time 10, yielding a higher life expectancy
- No extra-return on investments

Case c:

- New projected life table at time 10, yielding a higher life expectancy
- Extra-return on investments: 2%; 1.80% credited to policyholders (every year, through arrangement $f_t^{[1]}$)

Linking based on the number of survivors, indemnity based

t	$m_t^{[1]} = \frac{{}_tp_x^{\mathcal{F}_0}}{{}_t\tilde{p}_x}$			$m_t^{[2]} = \frac{{}_tp_x^{\mathcal{F}_t}}{{}_t\tilde{p}_x}$			$m_t^{[3]} = \frac{{}_tp_x^{\mathcal{F}'_{0,t}}}{{}_t\tilde{p}_x}$		
	case a	case b	case c	case a	case b	case c	case a	case b	case c
5	0.996	0.996	1.014	0.996	0.996	1.014	0.999	0.999	1.017
10	0.993	0.993	1.011	0.993	1.006	1.023	0.997	0.997	1.014
15	0.987	1.007	1.025	0.987	1.007	1.025	0.992	1.011	1.029
20	0.977	1.007	1.025	0.977	1.007	1.025	0.982	1.012	1.030
25	0.958	1.000	1.018	0.958	1.000	1.018	0.964	1.006	1.024
30	0.924	0.977	0.994	0.924	0.977	0.994	0.932	0.985	1.003

Linking based on survival rates, obtained from different life tables

t	$m_t^{[6]} = \frac{{}_tp_x^{\mathcal{F}_0}}{{}_tp_x^{\mathcal{F}_t}}$			$m_t^{[7]} = \frac{{}_tp_x^{\mathcal{F}_0}}{{}_tp_x^{\mathcal{F}'_{0,t}}}$			$m_t^{[8]} = \frac{{}_tp_x^{\mathcal{F}'_{0,t}}}{{}_tp_x^{\mathcal{F}_t}}$		
	case a	case b	case c	case a	case b	case c	case a	case b	case c
5	1.000	1.000	1.018	0.999	0.999	1.017	1.001	1.001	1.019
10	1.000	0.988	1.005	0.997	0.997	1.014	1.003	0.991	1.008
15	1.000	1.000	1.018	0.992	1.011	1.029	1.009	0.989	1.006
20	1.000	1.000	1.018	0.982	1.012	1.030	1.019	0.988	1.006
25	1.000	1.000	1.018	0.964	1.006	1.024	1.037	0.994	1.011
30	1.000	1.000	1.018	0.932	0.985	1.003	1.073	1.015	1.033

Linking based on assets and portfolio reserve.
Linking based on the actuarial value of the annuity

t	unfunded liabilities $m_t^{[9]} = \frac{A_t^f}{V_t^{[P]}}$			GSA $\gamma_t = \frac{A_t}{V_t^{[P]}}$			actuarial value $m_t^{[10]} = \frac{\mathbb{E}[a_{x+t} \mid \mathcal{F}_\tau]}{\mathbb{E}[a_{x+t} \mid \mathcal{F}_t]}$		
	case a	case b	case c	case a	case b	case c	case a	case b	case c
5	0.996	0.996	1.038	0.996	0.996	1.120	1.000	1.000	1.018
10	0.991	0.872	0.888	0.991	0.872	0.986	1.000	0.880	0.895
15	0.983	1.031	1.050	0.983	1.031	1.168	1.000	1.000	1.018
20	0.966	1.054	1.073	0.966	1.054	1.205	1.000	1.000	1.018
25	0.928	1.105	1.125	0.928	1.105	1.281	1.000	1.000	1.018
30	0.834	1.243	1.265	0.834	1.243	1.467	1.000	1.000	1.018

Total benefit adjustment up to age 95 & final value of assets

	$\frac{b_{95-x}}{b_0}$			$\frac{A_{\omega-x}}{A_0}$		
	case a	case b	case c	case a	case b	case c
no long. adj.	100%	100%	169.01%	-8.55%	-8.55%	8.93%
$m_t^{[1]} = \frac{{}_tp_x^{\mathcal{F}_0}}{{}_t\tilde{p}_x}$	84.53%	98.03%	165.69%	-2.98%	-7.58%	11.53%
$m_t^{[2]} = \frac{{}_tp_x^{\mathcal{F}_t}}{{}_t\tilde{p}_x}$	84.53%	99.27%	167.77%	-2.98%	-9.46%	6.55%
$m_t^{[3]} = \frac{{}_tp_x^{\mathcal{F}'_{0,t}}}{{}_t\tilde{p}_x}$	87.06%	100.96%	170.64%	-4.84%	-9.53%	6.38%
$m_t^{[6]} = \frac{{}_tp_x^{\mathcal{F}_0}}{{}_tp_x^{\mathcal{F}_t}}$	100%	98.76%	166.91%	-8.55%	-6.69%	13.87%
$m_t^{[7]} = \frac{{}_tp_x^{\mathcal{F}_0}}{{}_tp_x^{\mathcal{F}'_{0,t}}}$	87.06%	100.96%	170.64%	-4.84%	-9.53%	6.38%
$m_t^{[8]} = \frac{{}_tp_x^{\mathcal{F}'_{0,t}}}{{}_tp_x^{\mathcal{F}_t}}$	114.87%	97.81%	165.32%	-12.52%	-5.75%	16.32%
$m_t^{[9]} = \frac{A_t^f}{V_t^{[\mathrm{P}]}}$	72.66%	129.70%	224.52%	-0.49%	2.30%	24.81%
$\gamma_t = \frac{A_t}{V_t^{[\mathrm{P}]}}$	72.66%	129.70%	292.05%	-0.02%	0.18%	0.82%
$m_t^{[10]} = \frac{\mathbb{E}[a_{x+t}\|\mathcal{F}_0]}{\mathbb{E}[a_{x+t}\|\mathcal{F}_t]}$	100.00%	87.98%	148.69%	-8.55%	9.47%	56.73%

Some remarks

Main issues:

Due to the systematic nature of aggregate longevity risk, it is necessary to sort out the pricing and hedging problems

Appropriate policy designs may realize a satisfactory trade-off between risk and price of the longevity guarantees

In particular, it is important to match in a convenient way mortality/longevity with financial participation arrangements

Next steps:

Assess the cost and risk of the alternative designs

Solvency vs value created

Self-annuitization vs partial longevity guarantees, and timing of the annuitization

Thank you!

Contact: `annamaria.olivieri@unipr.it`

References

M. Denuit, S. Haberman, and A. Renshaw. Longevity-indexed life annuities. *North American Actuarial Journal*, 15(1):97–111, 2011

H. Lüty, P. L. Keller, K. Binswangen, and B. Gmür. Adaptive algorithmic annuities. *Mitteilungen der Schweizerischen Aktuarvereinigung*, 2: 123–138, 2001

A. Richter and F. Weber. Mortality-indexed annuities: Managing longevity risk via product design. *North American Actuarial Journal*, 15 (2):212–236, 2011

J. van de Ven and M. Weale. Risk and mortality-adjusted annuities. National Institute of Economic and Social Research. London. Discussion Paper No. 322, 2008. Available online: `http://www.niesr.ac.uk/pdf/290808_110826.pdf`